Logical Functions on the Real Numbers and Group Structure of the Irrationals

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Table of Contents

Introduction

This paper is about a system of logic, the foundations of which are continuous and discrete. As well, logical sets which are derivational of logical rings, functions, and fractals. These logical functions are sequential sets, evaluated in a recursive or dynamic fashion. We initially begin with some notation for these functions, explain how we arrived at these functionalizations, and then go on to describe their properties as well as derive some properties of their interrelationships. One end goal of this paper is to describe the properties of logical functions, ones which are capable of logically representing the geometric object the circle, as a set of functions convergent to the properties needed to define this object from base logical rules. Is the circle an identity in logic as it is very much in geometry?

We use a logical set of True and False, and adjoin Open and Closed, with appropriate logical rules. These, open and closed, as states, inherit a group property from that of the point. As well we can justify a squeeze theorem of logic at every logical state since we can define a continuous logical function. We can also prove many other properties of continuous logical functions.

Given the number of number bases these objects have their behavior is similar to that of rings. We construct these rings from number and set arguments, and find the number of ways we can obtain rings with these properties. These can be derived in one common way yet exist in profundity. We will find in this a natural isomorphism between systems of logic inclusive of open and closed and the extension of the rationals to the reals. This set theoretic approach offers other interesting results, because we can form conclusions such as forming an equivalence class of these rings with a free rotational symmetry.

This paper covers many of these topics, even delving into the number theoretic properties of these rings, strings, circles, or types as I will call them throughout. We will also touch on the ideas of these as fractalline numbers, and fractals, finding self similar sequences, as well as a fractal that describes the properties of the geometric object the circle.

A goal of this paper is to understand the connection between the operator's flexibility in transforming stings and the group defined by the logical operators. We would like the properties of the "operations" to be determined by the algebric group of these operators. Here, it seems that open and closed can alter the commutativity and associativity of a group if admitting these does so when true and false do not alone. With these operators we can find new and interesting results in number theory. Our construction accomplishes part of this, and the operator sequences are left with properties they get from the algebraic group of the logical tables we describe.

This connection, the number to algebraic or geometric, and the implications of this for understanding complexity in computation are interesting. For instance, this touches on Godel's theorem and Turing's work, as well as possesses a drive towards solution of recurrent, self similar, and other similar problems, that we believe have importance. An interesting question to ask yourself while reading is: "Is there an infinitely long consistent irregular and self-similar rule based sequence?"

Open statements are those statements for which there does not exist a logically definite underpinning until future declarations are understood, and are not defined as true or false, until all absolute truths or falsities are collected as pertains to them. Openness of a statement is therefore defined as a statement about which the truth has yet to be determined. This more expansive interpretation of open is 'undetermined'.

Closed statements in their absolute form are those statements that are not predetermined as either true or false, but for which further there is no openness and there is no declaration as to true or false. These statements are those that indicate they are neither openly true nor false and are essentially limited in application; in that these are closed to reaching a truth state. In this theory there exist bidirectionally open statements.

After developing a system of logic, we devise a one dimensional self referential system of logic by rules of inclusion, exclusion, and equality among sets, that is capable of producing simple results such as factorization and prime number generation. We look at those which possesses a simplistic number theoretic interpretation. Irrationals correspond to infinitely long self similar but irregular sequences, and are dense in the set of rings. However, we look at other questions, such as: Are there any consistent infinitely long sequences on $\mathbb{R}$, the real numbers, such that linear operations (natural number series evaluation) would never saturate the list of answered questions? It may be possible to encode nonlinear recursive processes with these strings.

We describe the geometry of the irrational numbers, and their group properties are similar to that of triangles or hypertriangulations. We examine in particular the transformation of a radical inverse. It is found that the rules of multiplication are somewhat isomorphic to division when we take the rationals to be the identity in the real number line interval $[0, 1)$. This construction is then a set a reals mirrored over the rationals, producing a set of functions with unique properties, only the irrationals possess.

This paper also attempts to address the following question: What properties of addition, multiplication, division, and subtraction are preserved under transformation by a radical? Mapping the radical to the vertical line in the upper half plane reveals part of the answer. Mapping to the radical as if over negative numbers reveals the other. These are interesting transformations, and reveal a greater depth to the properties of the real numbers.

This reveals a picture where the primes, under an inverse radical are the numbers for which squared means rationality. Since all numbers reduce to prime factorizations, we would think that the digit sequences in these irrationals, and potentially all, come from prime factors, as with natural numbers. However, we find the sequences of numbers become so long, these numbers cannot possibly saturate the list of irrational numbers in completion. There are many with differently characterized structures and qualities we cannot hence prescribe to the results only of the countable infinity of natural numbers.

And finally, the purpose of this paper is to study the way the pattern in the irrationals drifts off into randomness.

1 First Properties

As a brief introduction, consider some simple properties typifying those of these strings as in most systems that might be considered. We have a logical state defined by a character (ϕ), and then in turn by two others (ρ, κ). These two operations are of primary importance in the analysis of these logical sequences, or as I will call them, strings, because of their resemblence to rings as mathematically defined:

$$\phi = (\rho; \kappa) \tag{1}$$

The first of these is the state, the next two are the linear sum and the sequential product. Together, they define the logical function, and in two manners, one continuous and the other discrete. Both are summations of the information we have for these functions at a point in the function, the first being the sum of the states over an interval, and the second, being the group product of all logical operations along a statement. These operations are unique if we take them over a whole logical statement. These hold for all starting and ending positions as they generalize and possess uniqueness. First, we summarize some of their addition laws:

$$\rho_{r,t} = \rho_{r,s} + \rho_{s,t} \tag{2}$$

$$\kappa_{r,t} = \kappa_{r,s}\kappa_{s,t} \tag{3}$$

More accurately, to depict the construction of a given logical statii from other intermediate ones, we have many such intervals over which we may define the logical status. As we move through the string, we find that the current state may be represented in terms of those that come before and after, as with two statii. For two logical statii over two intervals, we have a whole representation in one function taken from these two determining function, (ρ, κ).

$$\phi_{r,s} = (\rho_{r,s}; \kappa_{r,s}) \tag{4}$$

$$\phi_{s,t} = (\rho_{s,t}; \kappa_{s,t}) \tag{5}$$

The concatenated logical function is then given by:

$$\phi_{r,t} = (\rho_{r,s} + \rho_{s,t}; \kappa_{r,s}\kappa_{s,t}) \tag{6}$$

Together these are the rules for composing functions, as well as making the two measures (ρ, κ) and the state continuous as given with many (uncountably many) logical statements, making the net function continuous. We can give definitions of the linear sum and the product as the following:

$$\rho_{r,t} = \frac{1}{\|t - r\|} \sum_{n=r}^{t} f(n) \tag{7}$$

$$\kappa_{r,t} = \prod_{n=r}^{t} f(n) \tag{8}$$

These carry with them the notion of using multiple functions together as a single object; the ability to have such a function that remains continuous; to have one which gives a continuation, and one with composition rules for it with other parts of the same function.

These are useful, because they can be used to fill in parts of functions. They have this property because they possess disjoint information about the logical states. We need both types of information to avoid potential many to oneness, to reconstruct the next element in a sequence, and to uniquely identify a string. This nonuniqueness is introduced by the presence of closed and open, which we will find to be useful mathematical concepts in the realm of logic.

2 Logical Ruleset

We can use the composition rule as a theorem for the potential continuation of a logical function to where it is not defined (forecasting), and backwards to before the function summation started, so as to have an entire, continuous, logical function. We can also use it to splice together different sequences. The only way we have not yet described is an interstitial filling. One can see that many of these transformations are equivalent or analogous to ones we would use with numerical type of series.

These both use a base function's values, the raw information of a logical nature, in the form of a set, with the function to be constructed. We have:

$$F(n) = \{f(n) : 0 < n < i\} \tag{9}$$

The logical set we work with is:

$$\sqcap = \{True, False, Open, Closed\}$$

Next, we have the logical tables for the values "True", "False", "Open", and "Closed", these include the union, the intersection, and the complement. Then, we have a table for operations we can perform on statements or rings. We generally use different tables for different purposes.

We have the usual logical tables, starting with the set theoretic union:

∪	T	F	O	C
T	T	O	O	T
F	O	F	O	F
O	O	O	O	O
C	T	F	O	C

The set intersection is different but also exists for these logial sequences:

∩	T	F	O	C
T	T	C	T	C
F	C	F	F	C
O	T	F	O	C
C	C	C	C	O

The complement is the not gate, and takes the other part of the interval. This is its table:

¬	T	F	O	C
	F	T	C	O

Then, we have the operator table, from left to right.

∘	T	F	O	C
T	T	F	O	C
F	F	O	C	O
O	T	F	O	C
C	T	F	C	C

These are the return values for two input values in the form of a table. This last table is composed of the values that we use in the logical function actively when it operates on a string. With logic we use the union to expand the potential end values of the string under use, rather than contract as with the intersection. Consequently we have two varieties of operation and two varieties of string. One variety of string is the state, and the other is the logical operation. We can also define other logical tables.

Generally, we wish to use the union to describe the composition of states, when the states encompass more set theoretic volume, given the inclusion of new states. We use this, when we want all states to be exposed. The application depends on the purpose. Additionally, we use the intersection, when we want to compose logical sequences as operations. The main goal of this is to orient our attempts at reaching a logical goal, as an intersection of alternatives, where the alternatives are a union of many configurations. The idea is to find a stricter conclusion as a logical intersection of rules on a union of states, when the latter expands with state number, and the former contracts with logical rules under composition. From this picture, we can imagine that not all systems contract and expand with the appropriate rates to reach a conclusion.

First, we must keep the logical operation of the string on the logical states, or list of True, False, Open, Closed values. The reason we do this is because the function must have a well defined range. Therefore, we let the function be given as the operation of the string on logical strings. With this a function is also an operator.

The result of a function is individual characters of a new string. When we do this we get a mapping. We use the function with the given set in a logical table, as an operator on another, or as an operator itselfon a ring. We do not get an identity but instead a new set of symbols. It is possible to form an object that has the behavior of a function and that alters a given function, in this system. These behave like the function itself. Some strings are derivable from others in a basis, especially via inverse functions, and some are not.

With these strings, the inverse string is the reversal of the normal string with true and false interchanged. One can see orientation is a different matter than an operation. The true false reversal combined with the orientational reversal, is a non trivial operation. This leads to a reverse string for which multiplication by the starting string is non-trivial. Results with these, as well as reversed strings, depend on the commutativity of the groups and therefore their order. This string however is the true false reverse of the last, and in general group multiplication depends upon order. In this system, we attempt to derive properties of our system from number theory to cycle back and produce new results. Group multiplication depending on order eventually leads to non-associativity as well. This property exists with the table for the complement and for the operator on the state. These are a state transformation of the nature of a reversal.

Although these are not the normal type of functions we will consider, we can consider functions of the nature of "T" $\rightarrow +1$, and "F" $\rightarrow -1$ as well. Usually we will consider functions that use more complex associations so as to compose rings. There is then the other way to identify a function, which is to take an if then structure, and identify a sequence by explicit cases of true, false, open or closed instead. As an example, we take the Cantor set and turn it into a statement.

$$\{CTC\} \tag{10}$$

$$\{CTCOOOCTC\} \tag{11}$$

$$\{CTCOOOCTCOOOOOOOOOCTCOOOCTC\} \tag{12}$$

The difficult part here is distinguishing the pointed end of logic from the blunt end of our system. The main things we can do is either include or exclude members of a set into groups, and consider comparisons of the nature of less than, greater than, and equal to.

3 Measures

One approach is the direct analysis of the strings produced. This can be very tedious and difficult. But, the tools that come from this are useful. Two helpful measures are the logically summative value, and the logical distance in statements from openness to truth value on average:

$$G(n_i) = \{< n_i - n_j >: \|f(n_i) - f(n_j)\| = 1\} \tag{13}$$

Open to Closed would be another result to attempt to measure. We also have the average absolute value of the logical summation. This is the average of the strength of the departure of the system from the closed state.

$$P(n_i) = \{< \| \sum_{n=0}^{i} f(n)\| >\} \tag{14}$$

The logical distance in statements from open to true and false states is analytic of the logical function, and the summative value is as well. These must both converge to asymptotic behaviors to produce a circle, if we are to take these to be logically natural and primary interpretations for behaviors of the functions. We then arrive at statements we must find the value of such as:

$$\lim\{< \| \sum_{n=o}^{i} f(n)\| >^{i<n_i - n_j>}: \|f(n_i) - f(n_j)\| = 1\} \tag{15}$$

This is a way of quantifying the modulo of the logic, as a pattern, the current $f(n_i)$ being the number of the state, repeated imaginarily the statements length over, until it flips. Building functions by counting True and False statements is a simple way to understand a sequence, but there are better ways.

How does logic quantify understanding? What is a circle in the context of a logical system? By creating one such circle by way of many things unrelated we create a very complex problem, whereby the result is unrelated except on a functional level, and the result is inadequate. Measures of frequencies in the set are measures, not operators. Instead of this process, we should use genuine functions, so discovered by a process of isomorphism instead. This is the alternative way.

4 Main Operations

We now develop a theory of logically based operations on strings. These infinite strings of True, False, Open, and Closed each behave as operators on other strings and on numbers. The composition laws are not those of numbers. We find that one of these strings can fit in the real numbered interval from 0 to 1, and act on others in the same interval. If we can set up the rules so as to preserve all operations then we can deduce groups of these operations. The trick to this is using operators with few logical bases.

If we can do this consistently, the results will be that of essentially a number system. We cannot use an infinity of different symbols, for we do not have a rule set this expansive. So, we use a minimal one of $\{T, F, O, C\}$. Since they have no common center, any point works as well as the rest to take their composition. This operation is the result of taking the string as a whole beginning at any point with the whole of another string, with an arbitrary center. The result is a new string with the no center property. This is a property of the strings, and not their particular sequences.

The two operations given before are important, and we have composition as before. To construct these other operations, recall that a logical string is defined by a logical set:

$$\sqcap$$

And by a function:

$$F(n) = \{f(n) : 0 \leq n \leq i\} \tag{16}$$

And for the set of all statements consisting of the true, false, open, and closed states we get the logical statement as:

$$\Omega = \{f(n_i)\} \tag{17}$$

Combining multiple logical statements can be done. Given a statement: Ω_1, and another, Ω_2:

$$\Omega_1 = \{f_1(n_i)\} \qquad \Omega_2 = \{f_2(n_i)\} \tag{18}$$

We form the net statement by the individual product of elements from each:

$$\Omega_N = \{f_1(n_i)f_2(n_i)\forall i\} \tag{19}$$

There are of course operations we can perform on a string. One operator is the set defined by taking every set of elements of a given length to a power of itself, to produce new elements. This is the power on all strings modulo some length:

$$\phi_l^p \tag{20}$$

Where p is the power and l the length. The logical table is ideally a set that preserves this modulo some integers. But we find that none of those listed do, besides the complement. Then there is the multiplication operator, by a statement P:

$$P(n) = \{p(n) : 0 \leq n \leq i \in \mathbb{N}\} \tag{21}$$

$$M(P, F) = \{P(m)F(n) : n, m \in \mathbb{N}\} \tag{22}$$

The identity consists of all infinite length strings that preserve another on the set of logical states. This is the closed on one end and open on the other flat interval from zero to one.

A string is both a function upon a function, and simply a function itself, in that it can be an operator on a function as well. Each operation is a string, and can be broken apart into m long pieces to produce new operations via the logical tables. One such function we can generate is also modular, it is the logical function taken in groups m long:

$$F(n, m) = \{f(l) : n \leq l \leq m\} \tag{23}$$

$$\{\prod_n^{n+m} \sqcap_n : \sqcap_n = F(n, m)\} \tag{24}$$

Finally have an inverse function:

$$f(n) \to f(n)^{-1} \tag{25}$$

This inversion is accomplished by reversing the logical arrow and interchanging or reversing all true and false states along the string. To reverse the arrow we flip the circle around as if rotating by 180 degrees. Together these comprise the main operations that exist upon the string, not including those basic ones that splice and remate the function to itself discussed earlier. We don't do so much to operate on the function itself on this level, but rather, with a function, and with other operations.

Derivatives can be accomplished, by looking at the function as an iceberg floating to the surface, in layers. First, with one derivative, we want to extract the essence of the function on a surface of one layer deep. A derivative is like the change in a function from one piece to the next. Using this as a basis we construct the derivative. We arrive at the following formula for the derivative, by finding that function which matches the changes from element to element in the list as given by the logical values of the logical operation. We wish to find the inverse with respect to the union, intersection, or operator. In other words, that operation that takes us from one point in the sequence to the next:

$$D(\Omega) = \{\sqcap \forall i : f(n_i) = \sqcap f(n_{i-1})\} \tag{26}$$

The integral is given by the inverse operation of the derivative. This is done by taking each one with the next. It is constructed with the state logical table for the union. This is the integral formula:

$$I(\Omega) = \{\sqcap \forall i : \sqcap = f(n_i) f(n_{i-1})\} \tag{27}$$

These strings may or may not have a repetitional structure in integration and differentiation. We can write these with a few relations. Abstractly, the reason is that the sample is one dimension shorter and hence for a first derivative is composed of individual elements. These strings are fundamentally operators. Higher order derivatives and integrals are given by repeating the process, and using effectively larger strings for the propositions P. We can complete each of these to infinite order.

With no center, the starting point on one string doesn't matter for these operations mathematically but does often in numerical practice. Equal rates of variability, and a whole string with a whole string are the rules for their composition. There is no center, in the sense of the product being independent of the starting location on string P or on string Q in PQ. This lack of a center property leads to multiplicative independence of the result with these logical compositions. With this; one can see a level of rotational independence is afforded by the construction of these strings. This is a composition of their sets, and they become logically operated 'types'. These 'types' are a new sort of number, and inherit properties of their group. Here is what this operation looks like mathematically for F, G and result H with a displacement of o:

$$H = G * F : \tag{28}$$

$$H = \{G(m)F(n) : n = [0,1), m = [0,1), \|n - m\| = o\} \tag{29}$$

Where F and G are their whole sequences, centered at n and m. The net composition is independent of o, yet the order matters, and in it, each element is the result of the operation of one entire sequence on an entire other sequence, as we move equally through both. Alternatively, we can describe this as the whole of one sequence taken with another whole sequence, with no lateral offset. The center being missing from both, is missing from their product. Although we would get a different sequence in general for a different offset, these strings have the property that we do not.

Finally, we have an operation that extracts the logical values for the function on all of a set of states by the set of logical values. For this, we use the propositions m in length $\{P\}$ as strings which operate on logical values unique to this system. This is the dual of the previous operation where we operated on the state with operations of length m. This is the operation of the function by modulo m sized pieces:

$$\Lambda_m F(n_i, n_j) = \{F(n_i, n_j)\sqcap = P\sqcap : \|n_i - n_j\| = m\} \tag{30}$$

This operation takes sections m long of the string as propositions P, and applies them to the four operators, creating a logical function, as if upon this silent set state operator.

These operations are then not entirely transitive, since most outputs go back to where their inputs were, or we have an integral displacement. With this, we preserve the structure of the base labeling over many operations, which is necessary for keeping the strings one to one when idempotent and for recognizing their identity.

This last operation is interesting, because it shows the logical function can actually be a multifunction on multiple inputs. By generalizing this behavior we truly have a continuous system of logical functions given that each depends on an infinite number of inputs, and each results in any of an infinite number of outputs. These do not necessarily correspond to numbers. These give a full way to treat logical statements in a system of continuous logic, but they require a level of expansion into new mathematical realms, to become fully continuous. We will not find ourselves limited by the small set of logical quantifiers, but instead by the process of counting.

5 Continuity and Symmetry

There are many natural symmetries of the number line. We use a few that are less well known than the first five in this list. These are the isometries as symmetries of the line we can work with:

1) Displacement.
2) Scaling.
3) Cutting into pieces. (Self similarity.)
4) Reversal symmetry.
5) Infinite thinness.
6) Rotational freedom.
7) Free range and domain input output scaling.
8) The structure of $\mathbb{R}$ exists over every interval size.
9) The numbers encompass a volume.

Many of these are fairly embedded in our minds. The non obvious ones are six, seven and eight. Six comes from the construction in this paper. A variable rate of covering (seven) is another continuous degree of freedom we will discuss. Finally eight comes from considering that in the mathematical realm there exist spaces where given the numbers are representation free, we find them without spatial interpretation, and consequently find that all numbers are somewhat contained in one another, hence all of $\mathbb{R}$ is contained in each point. This construction is one of reduced dimensionality, but it is also the limit of a neverendingly shrinking real interval.

A primary assumption to make this transition to a size of infinity may be to take the limit of the sequence size to infinity with these statements, but this is nonefficient and may leave behind structure. By doing so, every string is infinite in length, and all objects correspond to a system of continuous logic. We need merely then re-generalize and re-equip our facilities to handle all strings of this size. These carry over straightforwardly, and we find that the form of logic used is actually a subset of the more general system of logic.

When do we cross the gap between the discrete and the continuous, to get a continuous function, and when do we get a function defined with a natural continuation from the whole interval to the whole real number line? If there is no beginning to the logical state, can we still define a sequence without ends? An example, would be a closed curve such as the circle.

The strings become modal number objects. These resemble a more complex system of numbers than the regular numbers we are used to, and are in a more specific part of the mathematical territory of objects we could consider. These are in some senses but one of many general notions of number.

If we are to define logical functions as continuous, and are to reconcile the notion of a function used to represent a number, and these, we find that all sequences must be convergent, even if infinitely long, within an epsilon sized or infinitely small interval. This is so that over a continuum we find a continuous function. In doing so, we essentially give up on using the whole circle to extrapolate an algorithm, and revert to using sequences together.

These algorithms fit into these epsilon sized regions and although they are not contained in a point, they converge quickly enough for there to be virtually no volume over which they are not convergent. It will come in useful to have a constant function throughout in places. This is missing if we need room for the program and cannot be done but for the logical states, or the abstraction generated by a logical sequence operative of generating a constant.

By compressing the symbolic logic operation to a point, we can have continuous logical functions. To have a different function we need merely define each number in a sequence different by one in different logical values between two adjacent points on two functions. This is allowed as we have tremendous room to deal with a limit being taken even with such a change. The nature of the limit at every point must be the same. We need this capacity to define functions for which we have continuity completely with respect to the real numbers. Yet these are ones based on logical values.

We define a function as a mapping, but restrict the values of the functions to the logical states, and imagine one with rational spacing. These states are limited in domain to the regular pacing of the interval, so as to uniformly fill space. This restriction gives them a regularity for which we can construct a continuous curve. In as much as they are regular we can say they represent statements.

We can define numbers to these, not so much by a regular procedure of converting the sequence into a number, but by letting the sequence ask a question as to the nature of a number, repeatedly adding digits to this number in a logical process, by way of the averages of the sum of states and the product of states along the string or circle. Also, our strings can be taken to be rational numbers on the irrationals. i.e. $2\sqrt{6}$

These are not so much questions but are statements along the length of the construction of a number, and they give prescriptions for what numbers are about, in that they define infinite sequences whose results are as unique as the numbers so represented by way of an expansion of logical domain, and results that are exact in the infinite limit.

We can get the state 'closed', from comparing two outputs as a symptom of having two limits that never overlap between the two postulates, as for example one continually false, with the other continually true. It is desirable to act upon the larger logical context, wherein we have a supposition and conclusion, each of which may be true or false. We also have a logical supposition to conclusion that is reversible and true false interchangable. This should afford us two checks on the logical state.

This is the input and output of a statement, and should work for a reversed string, with supposition and conclusion reversed in action or direction of logic, but not logically reversed except as individuals in true or false value. The reverse within logic of the operators open and closed is closed and open. These reversed strings have many of the same properties as the others but are the true/false mirror image of a string and illustrate orientability is important for these strings. Whe one is true and the other is taken as false, together they are open or closed.

Open occurs when the statements do overlap, or commonly converge to a mutually true or/and false state or undetermined if sequential convergence is associated with convergence to a result, which it is not always. True and false eventually lead to a closed condition, and open disappears, leaving one of true or false. There are two logical gates, and supposition to conclusion reversal. Supposition to conclusion reversal is enforced by the symmetry in the logical gates as a sum, but not on the level of a direct symmetry, where we would have a symmetric logical operator. The logic operator is not commutative, but the solution works equally well from conclusion to supposition as in the reverse with as well $T \leftrightarrow F$ with these statements.

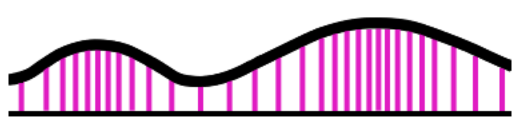
Running a system until we get a consistently closed condition after many true or false, leaves the majority open. Using closed as the start of a statement and open as the operation leaves closed for the conclusion. When not the closure of a state, but instead, acting as closure upon a state is one way to use these to have a controlled statement convergence, between postulates. Here, the rules of the logical set and the formula we use to create these sequences take on a more important role, for here they act as a ruleset in which to derive statements. This is not about a risky search for theorems but simply about deriving results of a number theoretic value.

There does exist a notion of correspondence between the functions as statements, and the construction of a number, or value in a sequence, by finding the closure or completion of a logical function over a closed interval of space. This is ideally suited to the circle. We use the function instead of over a flat interval, around a circle. In other words the sequence is ran around to connect to itself in the shape of a circle. It need not be a cricle but forms a closed non self intersecting curve. It is these places for which we must find a once recurrent function that has no beginning or end, for the circular types of these sequences.

The implications of using a variable rate input and output are interesting, for all that matters with these sequences and series is that they are ordered. With these we can accomplish resequencing of a sequence, and use the rings so developed to redefine the real number line. These are easily accomplished by associating two sets as a function with two different periods of coverage on the interval. One way to describe a function is by filling the interval with a series of density functions describing the distribution statistically and in relative proportion. To do this we will need a metric for the range of the function. The easiest way to do this is to have a variable density function. This allows us to understand the spread or proportion of T to F to O to C even when one goes to infinity.

We call the variable density source function a weight function, or a logical metric:

$$\omega(b) = N(b) \tag{31}$$



This is a depiction of a variable rate density in the range on the bottom in magenta, roughly tracking function height. Remember we are afforded this freedom given that only order of the rings and sequences is definitional of a seqeunce and not the displacement between two. This is important because we can do things like having a variable density to the range of the variables. This is different from an argument change and is essentially a move to a frequency function.

Such a lateral density function would afford us with two terms in the derivative, and can be done additionally by the encoding of another sequence. The analysis requires a reworking of the concept of limit and derivative, and a shift due to the properties these rings of rational reflection possess. These functions represent values in the non-discrete interpretation and are of a variable density in the sequence of irrationals.

The sequence, through rules, can be determined by these logical rules themselves, in that they can encode the order. These series put order before the notion of number, as the numbers to be encoded must, or rather depend upon being defined by their placement in sequences. Using the group rules and a spacing function as a series we can devise a law which displaces a linear sequence along its length. This can be determined by another sequence.

If all segments are chosen, but the order is determined by a new auxiliary sequence, then the modal rate is functionally variable. These functions are then in some sense variable in both their lateral and vertical degrees. Thus, we can have functions that always add to a continuously varying amount, even as it goes to infinity. These are a strange notion for functions, but admit a simpler inverse.

6 Number Rings

Every object in this system is both analogous to a number, in that it possesses properties like a number, yet it is not a number. I have called these rings, since their properties depend on the properties of many numbers working together, and they most resemble the mathematical object called a ring.

These are the properties of the rings:

1.) Asymptotically representative of number.
2.) Infinite number of elements.
3.) Non-repetitional.
4.) No center or initial starting element.
5.) Multiplication independent of displacement.

As a whole, these 'numbers' are really something of a 'modal' number, in that they have a modular structure given their rational base, are represented by infinite sequences which inherit the properties of a group under their operation, and reduce to numbers when we take certain limits. These are logical sets and behave as an object akin to a number fractal, or fractalline number.

These numbers, which I will call loosely, 'rings', have many interesting properties. One, that as we move through two series we encounter results along the way that compose a new unique product ring. This ring does not depend on the spacing or offset between the rings in this process but does depend on the order of the rings. Additionally, if a logical value comes up somewhere from a spacing, and not in the other, it comes up somewhere else with a different spacing. It will inevitably come up in the same relation to the composition ring that it had in the first scenario, but with a different initial spacing. This is so that multiplication is defined as without a center for the operation of ring composition.

In this the rings are independent of offset on the analytical level. It is a self similarity of the irrational number expansion under rational multiplication groups that causes this. We sacrifice this additional property of different results with spacing, for the property of multiplicative independence under a ring ring composition. As a side effect we get a multiplicative type of group isometry. These functions inherit the group properties of the logical group they typify. These are polysynchronous in that they each are actually made up of multiple periods, and expose these numbers under different operations or upon rings. These all possess a group with multiplicative like properties but without an identity.

An argument that we can define such sequences, is given by the hypothesis that they be complete. That if one element was missing, we could find it given the rest of the sequence, so that it would possess the properties it does, most especially, no center. Given this one piece and duplicates, of this existing, we can therefore find the whole sequence in principle with enough copies, or define it as unique. With this other part of the sequence present, we can use one to operate on the results of the other, and get a completion with this missing piece from the rules inferred from the completion of the ring with the piece missing.

An example is one that is open. Here we merely mirror the elements to get a sequence:

$$OCFTTFCO \tag{32}$$

One part is then the unidirectional complement of the other and the strings resolve to a single logical element. With this process, it is true that if we have a missing piece, and can fill it in uniquely, then there exists a non-constant ring in comparison to this position on the ring. Noting this, we must have a way to combine rings independent of an origin, since the sequences have this property.

We can uniquely produce infinite one dimensional product rings from two one dimensional ones in the manner given before for their multiplication, with an independence on o. These functions or logical sequences exist side by side, yet do not depend on a center, or in their displacement in multiplication. From this, they are polysynchronistic in the number series. We have for "multiplication" of these:

$$H = G * F \tag{33}$$

$$H = \{G(m)F(n) : n = [0, 1), m = [0, 1) : |n - m| = o\} \tag{34}$$

We also have special cases:

$$\delta(n) = \{G(n + 1)F(n) : n = [0, 1), m = [0, 1)\} \tag{35}$$

$$M(n) = \{G(n)F(n) : n = [0, 1)\} \tag{36}$$

If (in the continuous limit) we can find a solution where:

$$M(n) = \delta(n) = \Delta_m(n) \tag{37}$$

Then we have a sequence without center, under multiplication. Or in other words, a result independent of the displacement between the sequences. This requires two sequences to define. A way to do this is with sets of numbers with properties and divisions into the appropriate sets in the interval. These may have a variable density. The idea is to set them out uniformly so that there is no center in reflection over any of the points we use as starting points in the multiplicative operation. For this we will need the full rationals.

We consider P uniformly distributed mirrors. These reflect off each other in such a manner that they become dense. By the extension of mirrors as $1/P$ in size and in Q parts we find mutual symmetry and equivalency to: $\frac{Q}{P} \in \mathbb{Q}$.

These are the reflection points, and constitute their own image in the way they reflect over themselves. There is no rational center among the rationals in the unit interval. They are identical in all the rationals. Also, although this will not be required, these occur at an even scale division. This allows them and their multiples (all of them) to cover the whole interval. With a ring as the identity of these mirrors.

These can be reduced to one of many kinds by way of an integrally based logical representation:

$$\frac{odd}{even} : Open, \frac{odd}{odd} : True, \frac{even}{odd} : False \tag{38}$$

This particular representation may be interesting, but for now, the algebraic connection is not clear. We need more elements of this group to establish a one to one relationship with a function. We can, of course, come up with an alternate prescription such as:

$$..., \frac{p}{q} : O, \frac{p+1}{q+1} : C, \frac{p+1}{q} : T, \frac{p}{q+1} : F, ... \tag{39}$$

This and other sequences, afford us great freedom, so long as we distribute them evenly, but the correspondence to the natural numbers is not clear. As well, because these are rational sequences, when we include the rational numbers for every element of the set, the sequence becomes folded over into itself such that we have repeating locations in the reals. What we need abstractly is a set of numbers for which this pattern of all integers over all integers is preserve,d and not duplicated, so that it is unique. For this we must go to greater depth, to that of all rationals in each member of the sequence.

This is difficult, but can be conceptualized as an infinite set or infinite series, convergent at points along the real line. This is abstractly the multiplication of two sets. The interesting thing is we can imagine fixing the rotation of the series (its slope) into a set of places finite in size; to keep them in alignment. But, in this scenario what really happens is the gap size between mirrors goes to infinitely small, and the sequence lives here, existing only within the limit, the gradations in the functions now being continuous. These are many in kind, and do have roughly an order, given by the properties of the end limit sequences which are infinite in number. With this, the sequences have no center, but they have an order, holding for any two in multiplication such that the result is unique and also without a center.

If we have all rationals: $p/q = r \in \mathbb{Q}$ then these satisfy a rotation property. Consider:

$$\frac{p^*}{q^*} = \frac{p + m \cdot o}{q + r \cdot s} \in \mathbb{Q} \tag{40}$$

With:

$$m, r = 1, s, o = \{0, 1\}, p, q = 2k, k \in \mathbb{N} \tag{41}$$

We get the rationals. These function by themselves as one element, but the idea now is to look for a group of their elements that behave like the rationals under this set condition, and such that in order, produce a sequence freely for the interval. We must take all numbers divided by all numbers for all mirror distances. This insures that we have a function of a form such that all reflections of all reflections, of the sequence elements as numbers, give equivalent series for all such numbers. With this we have a centerless product, given their construction. These only work when 'rotated' by a rational number, but, we have multiplicative independence on displacement.

The general representation of a rational number looks like:

$$\frac{p}{q} \in \mathbb{Q} \qquad p \in \mathbb{N} \qquad q \in \mathbb{N} \tag{42}$$

With q scaling with the numerator such that $p < q$, this spans a group and has no center. The p are now different elements of this ring. But this causes a further problem. The elements overlap in the ring and cause interference. From this, we must resort to an alternative. A member is now an irrational.

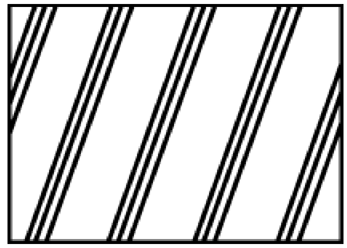
Every irrational number is duplicated into all rational numbers for every element of the sequence, since the rational numbers have this mirroring property and no center we require. By doing so, we learn that the sequence, if not duplicated in its rational multiplier, must consist of all mutually irrational numbers, with the exception of one rational point. This construction is actually a return map of all irrational numbers mod all rational numbers. This has the required properties, and every one of these elements, in a list, must be an irrational number. There are enough of these to fill the required space.

If we did not go to the irrationals, and used the rational points as the complete set, instead we would get all the same symbols, and in being the full set, it would not be able to symbolize different symbols uniquely. They are therefore not a suitable basis by themselves, and need extension into the irrationals.

Once we have broken free from the shackles of the rationals the irrationals form a new basis for the set. Modulo all of the rationals we can produce a function with the multiplicative property needed, as everywhere the sequence is then mod the rationals, and a limit function reveals a starting sequence. All we can do is probe the surface of these functions. The way we order the elements is roughly by the slope of the limit function on approach. It does not matter how we choose points if we let an irrational sequence approach a limit by reiterative tiling, so long as order is preserved. By geometric construction we can show that the numbers in a given exact order of rational multiples in the real interval from 0 to 1, fall in order at increasingly later times with couniform transport, rather than discordantly. This means there exists a similar sequence in the irrationals on every scale such that they map to the same end point with their symbols in order. We see self similarity in the graph of their multiples.

These are the conditions for elements now:

1.) No duplicates in the set among $\mathbb{Q}$.
2.) Each an irrational potentially multiplied by a rational.
3.) A different rational among the set with each one.



Depiction of irrational slope passing through a tiled rectangle an infinite number of times before meeting itself again.

For this diagram, when we have rational measures of an irrational number difference, it determines order. In repeating this process as we go in order the numbers appear to the larger side of a given rational as a repeating pattern, still in order and from smallest to largest. This is one absolute form of order. To make a sequence that is in order of irrationals, restart when modulo the rationals at some distance; $1/N$ in practice, ahead of the starting irrationals and go in a circle. The numbers you resolve will be the irrationals needed in order to produce a convergent sequence. In fact, each element of these sequences as a whole are in an equivalence class, and so therefore obey the properties of a group.

These are in symmetric juxtaposition to the rationals, and among the real numbers, but we must use all of an infinite sequence to construct them. Given their property to fill in all the gaps between the rationals, the irrationals form a complete complement of the rationals, and are mirrored over one another, and to only depend on orientation. These numbers are therefore at the right locations to be duplicated via the rationals, to fill a dense set as reals. We cannot use the rationals as one such set, because they self overlap and we can construct a sequence only with them. This explains the absence of a multiplicative identity; it becomes $\mathbb{Q}$. We use the irrationals each times all of the rationals. Then our numbers are rotationally free of the rationals.

At least one ring exists, the null ring, consisting entirely of zeros, corresponding to the included point of zero in both the irrationals and rationals. It is here they overlap. Since the rationals cannot reach this element it is not a part of our construction. It is however reachable with sequences and as an element of the reals. We can use it as a ring, but now as an element strictly of the reals, and not the rationals. The system is strongly determined by its first element, but for the most part, a sequence appears chaotic. There are many ways we can associate a sequence with an irrational number.

The expanded set is all:

$$a = \frac{c}{d} \cdot b \in \mathbb{R} \tag{43}$$

$$\frac{c}{d} \in \mathbb{Q} \qquad b \in \mathbb{I} \qquad \mathbb{I} = \mathbb{R} \backslash \mathbb{Q} \tag{44}$$

Although the real numbers are required to fully resolve properties of these numbers, we can rudimentarily use rings with a rational set of values for analysis on a computer. We do require the irrational numbers, as the rationals as a whole are but a point in the irrationals. For these we need parallel constructions of rational numbers multiplied by irrational ones, over themselves as an infinity of layers or tesselations. With the rationals, the irrationals, complete the picture with the real numbers. We need this full set of numbers along the real number line to do this analysis.

It is not wise to space the numbers at regular intervals, because then by the rationals they cover each other, and we get interference. With purely irrational numbers we can use irrational numbers in the interval with an irrational spacing. But, then we lose order and the unique meaning of the symbols, which becomes important as $N \to \infty$. We cannot use irrational spacing again because we produce a rational number inadvertently. If the irrationals are different with rationals, we can avoid rational number overlap. So, these sequences are sequences of irrational numbers each times a rational number. These will preserve order at all scales. We will find mutual ring agreement is the foundation for a limit.

The irrationals tile over themselves an infinity of times before matching up to themselves. In this case we have two infinities in these numbers to contend with. The irrational, from moving over an infinity of times to match up, and the rational, by covering with a countable infinity. We can make use of this flexibility to construct these new objects as numbers with an isotropic multiplicative identity. These two types of infinity are used in different ways.

This process forms an infinitely numbered extension of the rationals by the irrationals. Each element is actually the set of all rationals multiplied by each element. This makes up an infinite set of rationally displaced irrationals. Rational points alone would not work, as they overlap with other sets of rationals, but, irrational points do work, as long as the elements so adjoined are all mutually irrational (under multiplication and division). These may disagree or agree in their end point. No single radical base works with all integers, since these overlap with squares and thus the rationals. Since sequences of irrationals quickly become rationals and we have few other tools available, one may speculate these are really transcendental numbers. We do have non-algebraic numbers in this set.

To get an irrational sequence convergent to a point consider making regularly spaced irrationals at rational values to preserve order. What we find is that then the rationals overlap, and our desired property is lost. This can be relieved by considering sequences of irrational value, of differing values. These irrational numbers should be all different. Later we will add a multiplicative constraint for which their product cannot be rational. One such sequence is convergent. We find the elements of this type of number ring or sequence are:

$$\mathbb{Q}[\{r_n \in \mathbb{R}\backslash\mathbb{Q}, n \in \mathbb{N}\}] \tag{45}$$

These r_n symbolize elements of a sequence. If we take the collection of all such objects we get, remarkably:

$$\mathbb{R} \tag{46}$$

We construct these series by adjoining a set of r_n, each times all of $\mathbb{Q}$. The properties of these as numbers are that they have a natural number labelling. We generate one of these sets by an irrational sequence multiplied by a rational sequence or not. At most, one element of the sequences as a whole, can be a rational number. This single rational number must therefore be possessed by only one sequence in repetition. This sequence is the zero sequence, the additive identity.

The multiplicative identity has become $\mathbb{Q}$, of which one element, the zero sequence, is shared between the sequences and rings. This is the continuous open sequence as well. The generator of the multiplicative identity, is actually the additive identity. The collection of all of these rings behave as the real numbers, $\mathbb{R}$, because zero, is representative of all of $\mathbb{Q}$. Yet, they are really a series of numbers. If we use a unique rule for all such series then these are an infinite sequence of irrationals yet they may be multiplied by a series of rationals:

$$\mathbb{Q}_{sub} = \{q_n : q_n \in \mathbb{Q}, n \in \mathbb{N}, q_n \to q^*\} \tag{47}$$

$$\mathbb{I}_{sub} = \{i_n : i_n \in \mathbb{R}\backslash\mathbb{Q}, n \in \mathbb{N}, i_n \to i^*\} \tag{48}$$

We find that the multiplicative property is now directed at irrationals, and addition to rationals. For the product this is a return to the identity from two irrationals, while for the sum the identity is rational while the numbers we operate on are irrational.

The following is then the object we have constructed, and many look this same way. To obtain our symmetry with respect to the rationals, these must be convergent to an irrational or rational number. This requires that both $\mathbb{Q}$ and $\mathbb{I}$ sequences must converge both to rational or irrational numbers for a mapping back into the set.

$$\mathbb{B} = \{o \cdot q_n \cdot i_n : \forall o \in \mathbb{Q}, q_n \in \mathbb{Q}, i_n \in \mathbb{R} \backslash \mathbb{Q}, n \in \mathbb{N}\} \tag{49}$$

The limit of $\mathbb{B}$ is the whole real number line. We hope that:

$$\mathbb{B}_{seq} \to q^* \cdot i^* \tag{50}$$

We get a result of:

$$\mathbb{B} = \{\mathbb{Q}[\{r_n \in \mathbb{R} \backslash \mathbb{Q}, n \in \mathbb{N}\}]\} \tag{51}$$

This has elements r_n such as:

$$\frac{1}{4\sqrt{37}} \tag{52}$$

In other words rings are from:

$$\mathbb{B} = \{\mathbb{Q}[\mathbb{R} \backslash \mathbb{Q} \times \mathbb{N}] \not\equiv \mathbb{R}\} \tag{53}$$

The notion that we may multiply without a center implies this is a problem of the nature of multiplied numbers in classes, two of which produce one under $\times$. Starting with the previous set we take:

$$p, q \in \mathbb{B} \tag{54}$$

To show:

$$p \cdot q \in \mathbb{B} \tag{55}$$

And:

$$(p + l) \cdot (q - l) \in \mathbb{B} \tag{56}$$

And:

$$p \cdot q = (p + l) \cdot (q - l) : \forall l \in \mathbb{Q} \tag{57}$$

We take:

$$p, q \in \mathbb{Q}[\mathbb{R} \backslash \mathbb{Q}] \tag{58}$$

As:

$$a \pm b = a \in \mathbb{B} \pm b \in \mathbb{Q} \tag{59}$$

Then we have N natural number sequence equations:

$$p \cdot q = (a \in \mathbb{B} - b \in \mathbb{Q})(a \in \mathbb{B} + b \in \mathbb{Q}) \tag{60}$$

Or:

$$p \cdot q \in \mathbb{B} \tag{61}$$

And we find:

$$-l^2, l(q - p) \in \mathbb{Q} \tag{62}$$

Then, these rational parts vanish when we subtract out $\mathbb{Q}$ if we think of it as a fixed set, leaving:

$$p \cdot q = (p + l) \cdot (q - l) = p \cdot q : \forall l \in \mathbb{Q} \tag{63}$$

We can infer that, given:

$$p, q \in \mathbb{B} \tag{64}$$

Then:

$$pq \in \mathbb{B} \tag{65}$$

Now we can say:

$$\mathbb{B} = \{\mathbb{Q}[\mathbb{R} \backslash \mathbb{Q} \times \mathbb{N}]\} \not\equiv \mathbb{R} \tag{66}$$

Note that $l \equiv$ the multiplicative identity. This means, anywhere a multiplication is, we can introduce or remove a rational number. As well, we may treat a rational and the multiplicative identity equally. This indicates that the rationals are taken to all have a value of one in multiplication.

We learn that generally, we can multiply out the rational parts. This is interesting, for it clips off all but the tail of an irrational number. Lastly, this holds for the whole sequence, showing that if we take two of these objects we can produce a new one, in the same set, for which there is also no center under multiplication. What we have just shown is that multiplication is rationally independent. Is this the only such group to behave this way?

With the sequences encoded into the r_n, we can say that every such sequence is non-repeating. We have obtained our second property. The first was multiplicative independence, which we have met by construction. Each r_n brings an entire $\mathbb{Q}$ along with it. Yet, each r_n is not an element of $\mathbb{Q}$ but one of $\mathbb{R}\backslash\mathbb{Q}$. These numbers are irrational. Although the answer does not seem clear cut. A series of irrationals can converge to a rational just as a series of rationals can converge to an irrational.

The order is not always precisely clear, when using different irrational numbers purely, however it is when we use a sequence of irrationals convergent to a point. If they are convergent to a rational point, they spend most of their time in the neighborhood of a rational, and have one such infinite element. This may depend on the point. We can have an element converge to an irrational as as these may represent sequences.

These groups have no center, but we find they are only rotationally free at the level of $\mathbb{Q}$. To get them rotationally free at the level of $\mathbb{R}$ and still contain information is interesting. This would however mean there would be more numbers in $\mathbb{R}$ than there are in $\mathbb{R}$, which is not possible. So we stop at rotational rationality, which corresponds to these spacings. This group of objects is interesting, and not null. We can arrange the strings to be any sequence we like, following one of these irrational spacings to the limit of a series of such numbers. These hence do not represent numbers literally in every context, for example they could be taken as sequences to be re-interpreted as numbers. These numbers are consequently irrational.

Their behaviors, if each number is a ring or vice versa, are that of the real numbers. Only these possess numbers representing the full breadth of the sequences with rationals, that fill the space with a sequence. This seems to suggest that these properties are those of the real numbers. But these are each but a subset.

Every one of these is an irrational number modulo rationals or a rational times an irrational. It is simply a natural number counting set of all the irrational numbers specified. The interesting thing is that these imply sequences each of which contain no repetitions. For this, the sequence must be everywhere unique, and this is a very interesting property, because the numbers are entire in their irregularity, wherein, considering any bases of the ring, we can arrive at no repetitions of this base the ring over. This proves the property of all rings being non repeating. With a rational and irrational as sequence elements the elements can be elements of other sequences or can be other sequences in general. However they may not be such that they overlap, except when

With an implied convergence with ring length approaching infinity, it is sufficient to take large rings for statistical purposes. The pattern is given by a sequence of T,F,O,C as a string in its order. For instance the last gives a unique string of logical elements. This resolves to a set of functions and set of irrational numbers, but the above series sets the order. Irrationals preserve distance and therefore order with the tiling of them into adjacent squares across the interval when we use them in a convergent series. With this, we can use any sequence which does not overlap with the rationals. We can actually technically use one rational point per full set of these rings, as any more would create two points in the series that overlap, implying the existence of a rational relationship between them and a multiplicative identity.

We find that rather than encoding for each of the logical outputs we can merely use a counting process of natural numbers and irrational numbers as elements for the sequence, $F(n)$ when we fill the circle with all multiples for a given item in the series. The $F(n)$ is now a function on these regions of $\mathbb{R}$, with any regular sequence we like, beginning from anywhere, equally spread out in a circle around the circumference. This construction is such that there does not exist a "zero" or start in the sequence between two circles and multiplication is independent of relative angle.

The complete collection of all of these numbers is given by:

$$\mathbb{B} = \{\mathbb{Q}[r_n] : \forall n \in \mathbb{N}\} \not\equiv \mathbb{R} \tag{67}$$

$$r_n = a_n b_n \tag{68}$$

$$a_n \in \mathbb{Q}, b_n \in \mathbb{R} \backslash \mathbb{Q} \tag{69}$$

A sequence of rationals in the irrationals would spoil the symmetry between what is reflected and what is real. Consequently, there is a single rational in $\mathbb{B}$. As these numbers exist they are in some ways two dimensional. For they are the composition of two different series: a rational and an irrational, and these are each infinite in number. With this we can put them into a two dimensional correspondence with $\mathbb{Q} \times \mathbb{Q}$. This object has the series numbers, but not their values under logical evaluation. If we are looking for interesting number behavior we should examine the connection more clearly.

When we compose these rings, we find a correspondence of the nature that when we take the two series and consider their limit point, it must be a symmetry point of the rationals. Therefore the limit point is not a new sequence. If we put these on the same footing however, then we should have no problem. The rationals take care of this. They produce a seive, because the remnants are the part that distinguishes the rational from the irrational, and only the structure of these is left over.

We can delimit the first rational part, but soon we are left with no number. These numbers however are irrational, and we can not delimit a part to reduce to the rationals by multiplication, addition, subtraction in general. We do not divide by these numbers in this process. Instead of an element being missing alone, it is also missing from all rings. Or, it is present in the form of one statement of neither true nor false.

For this consequence it is neccessary that there be an exclusively new element or category for which a labeling system of construction of this sytem, (given its uncountable nature) requires a new element to be associated with these numbers and with this individuated rational ring representative of all.

These sequences under logical operation are nonassociative and noncommutative in general. Finally, when the decimal part becomes a fractal we have a fractalline number, and it should display this behavior at all scales. Is such a number derivable within this system?

The primary impetus of this construction was to obtain no two equivalent numbers for in every series. With this we obtain a circular continuous group with one multiplicative identity element. These are the rings and the sequences.

To do this, recall we are working in $\mathbb{R}\backslash\mathbb{Q}$. This is with the exclusion of $\mathbb{Q}$, which includes 0 and 1. These are the usual additive and multiplicative identities. The elements 0 and 1 are asymmetric. One is present and the other is absent from this ring. One is not included but as a limit point, so it cannot be the multiplicative identity. Also, for each sequence all the a_n are distinct and all mutually irrational as can be found by multiplication. The additive group stays as normal, meaning we can still have an additive operation and even an additive identity of zero, which we include in the set for this reason. It appears to be null. It does nothing under addition, yet can be a result, and is the one rational point. Ideally we want to construct these series by:

$$\mathbb{S} = \{a_n \in \mathbb{R}\backslash\mathbb{Q} : \forall n \in \mathbb{N}, a_n \notin \{\mathbb{Q}[a_i] : i < n\}\} \tag{70}$$

These are then all irrationally related irrationals. There are no rational multiples among all of these, and all numbers are irrational. As well we can see this set is not complete, because it 'comes into being' as the elements are added. We need this for the multiplicative structure. As well, the sequences must be rational or irrational to satisfy the property of functions to possess elements of sequences.

This set can be constructed, and is unique when they are placed in order with rational multipliers. Yet, there is no natural way to do this, and as a consequence we have no ordering but the one given. These provide the ordering of the numbers but not the sequences convergent to these. These limits are therefore unique but not for every unique sequence. We expect that the irrational digits converge as fast as rationals. This is likely not the case, with summation taking different amounts based on the irrational. This gives reason to treat such sequences as equivalent. Only then are they unique results, but the inverse can never be with points, excenpt an irrational. This level of meaning depends on the level of our ability to decompose a number.

These sets interestingly enough are convex, as the irrationals are and although they diminish the set of the irrationals, they as a set expand. There are uncountably many infinite valid elements that can be added. This gives the impression of an expanding realm of application, but these vectors alone are very low dimensional. However, the chance of mistaking two numbers would be very low, and due to accuracy only. There is ambiguity however because a number could be a rational multiple of another irrational number, but we may not know which one. One would think the irrationals would mask such a regular pattern. But, the pattern itself hase resiliancy, and we may be able to deconvolve it. There are however rational sequences within the irrationals. We find the rationals are to the modulo classes of the integers, as the irrationals are to the primes. They are in an equivalence class of irrationally related irrationals.

In summary, the rings are the numbers in this theory. They consist of irrational numbers and as statements are equivalent to numbers. They have a multiplication property, but not by rationals. They have an additive identity, and may be a rational amount from another irrational, but not a multiplicatively related irrational. This preserves the structure of having no center to the product.

The irrationals are rotationally free in their group. And, in existing without a factorization, they are independent of the fundamental theorem of arithmetic in the group of their digits. Consequently they do not usually have a pattern. But, this independence alone gives them a freedom to exist with a group nature of their own. This group structure is cyclic, as we have found, among their sequences. We can form any combination we like, and the order of the these elements does not matter. These form a commutative group.

We obtain the full structure of $\mathbb{B}$ by bringing in all products of all Ω_P:

$$\alpha_P = \{r_n = \frac{\sqrt{P}}{\sqrt{Q}} : n \in \mathbb{N} : P, Q \in P^*\} \tag{71}$$

$$\Omega_P = \{r_n = \frac{A\sqrt{P}}{B\sqrt{Q}} : n \in \mathbb{N} : A, B, P, Q \in P^*\} \tag{72}$$

$$\mathbb{B} = \prod_{\forall} B \qquad B \subset \alpha_P \tag{73}$$

Is it true that the structure of all possible irrational sets produces or is equivalent to the products of the reduced group Ω_P and that all such possible sets is the full set $\mathbb{R}$? Do these duplicate the structure of the primes or natural numbers?

This appears possible, given the existence of numbers such as $1/\pi$ and $1/e$ which are transcendental and yet could be the limit of an algebraic sequence such as these.

We have learned there are several types of numbers we are dealing with:

Irrationals: $\mathbb{R}\backslash\mathbb{Q}$: All irrationals in $\mathbb{R}$.

Rationals: $\mathbb{Q}$: All rationals in $\mathbb{R}$.

Irrationally Irrational: Numbers not a rational multiple of any other in the set. These are neverending irrational sequences. We cannot take from the outset of the selection of an irrational random number for it to be fundamental, or a base prime irrational. Instead, we see continuum irrationals to be made up of many irrational bases, with potential multiplication by rationals. Those elements selected from $\mathbb{B}$ are suited to this task, and fundamental. Those elements selected from Ω_P are suited for this task.

Irrationally Irrational Cosets: Despite the ambiguity in the irrationally irrational numbers we can still define these in a constructive manner. These are those numbers which are rational multiple cosets of a given irrational. They are not unique to the base irrationals given, since we could multiply two and would get a rational prefactor on another base irrational. They come out with other rationals in this process. But, we can with twice over multiplication produce a rational and by matching up to the now rational part we have the other rational part. Those elements selected from Ω_P are coset members of those from α_P. Is there a coset process that terminates in all irrationals, or should we properly define this as but one step?

7 Polynumerous Types

We should not get the identities of the logical system and the groups of numbers confused, one is of a much larger measure as compared to the other. Although the logical matrices have a measure of zero relative to the sets, these operators have identities in the ring, as well use three states together, so these sequences become higher dimensional in more complicated logical structures.

These are the properties of sequences:

1.) Asymptotically representative of function.
2.) Infinite number of elements.
3.) Every element a member of the logic table.
4.) Symmetries and group properties inherited from logical table.
5.) No center or initial starting element.
6.) Operation independent of displacement.

In:

$$\mathbb{B}_{seq} = \{q_n \cdot i_n : q_n \in \mathbb{Q}, i_n \in \mathbb{R}\backslash\mathbb{Q}, n \in \mathbb{N}\} \tag{74}$$

$$\mathbb{B}_{seq} \to q^* \cdot i^* \tag{75}$$

The sequences give different outcomes as a result of string string correspondence. When we can derive an irrational number of the same kind we get a matching condition, and they correspond. When we do not we get other cases. The rationals are the space we design our logical functions around. Consequently, they are the space where our produced string must be close to the real one. The irrational space is the space of our logical system, and where the statements are contained. There are four cases of convergence to consider as a result:

ring-ring comparison	same seq.	different seq.
truth value	T	F

The ring ring comparison test.

operator on op.	$\mathbb{Q}$	$\mathbb{R}\backslash\mathbb{Q}$	sequence/ring	converges
same # set q^*,i^*	O	F	s	Y/?
different # set q^*,i^*	T	C	s,r	N
sequence/ring	s	s,r		

The operator table, for operations of these upon each other.

operator on ring	$\mathbb{Q}$	$\mathbb{R}\backslash\mathbb{Q}$	sequence/ring	converges
same # set q^*,i^*	C	T	s	Y/?
different # set q^*,i^*	O	F	s,r	N
sequence/ring	s	s,r		

The table for an operator on a ring.

operator truth	$\mathbb{Q}$	$\mathbb{R}\backslash\mathbb{Q}$	sequence/ring	converges
same # set q^*,i^*	C	O	s	Y/?
different # set q^*,i^*	F	T	s,r	N
sequence/ring	s	s,r		

The operator table, logical state of one operator to another.

We can explain the reasoning behind this as follows: 'Different' corresponds to the sequence having members in different sets or if we were to change the order in the table. We have several types of action. One is an operation on an operation, two is an operation on a ring, and three is a ring on a ring. These give different results. If the irrationals are considered unique, then it is clear that they only return equality under this limit series when they match up to identical copies of themselves or produce a rational. With this there are two types of information contained in every number. We will find this ring analogous to a ring of primes.

When both elements converge to rational numbers, what we obtain is considered outside the ring, so it is a result, and must be either false (rationals different) or true (rationals the same), since we have a truncated statement, it does not pertain to closed or open. Recall the rational numbers are the identity. When both elements converge to irrational numbers in a test for equality, because the two infinitely long statements yield a series that either multiplies the same way and is mirrored (such that the product is rational), or we get a new irrational number. We get true for a result, open, for a ring, and true or false, for an operator on a ring.

When the first is convergent to a rational number and the second to an irrational number, the point is not an element of the ring as of yet, or has yet to be obtained, and the means are not subtle enough to uncover the irrational number, so the result is open. When the first is convergent to an irrational number, and the second is convergent to a rational number the logical operator is open but the logical state is closed. This is therefore false for this operation on a ring.

The presence of true and false and open and closed gives us an operator string, because these values instantiate such behavior, and, strings that are entirely open and closed are reserved to be rings. We take irrational numbers themselves to be equivalent to long strings of true and false or open and closed, when we do number correspondence. If we can show a one to one correspondence between the series going into statements and a produced point then we have a view into the behavior of the decimals of irrationals.

This process of inclusion and exclusion on the set seems very reasonable, given that we identify an element against a background. As well, we now have comparisons. If the elements of the series are never rationally related, then we have identified two completely unique numbers, with the property of a universal freedom over the rationals. These possess and maintain irrationality with respect the rationals under multiplication as a group.

We must decide which logical purposes values in the strings and the operators possess. This is neccessary, because without it we will not have a complete dictionary for the numbers. Also, we can simply use $\mathbb{Z}_4$ but this produces an uninteresting dictionary and specifies no boundary between operator and number.

What we need is a correspondence between the logical operators and states in the operator strings and the state rings. The operator strings can act on each other, while the state rings act like numbers on each other. For this, we need to be able to somehow represent state multiplication and addition. The operators meanwhile operate on the state rings to produce new state rings. The operators should operate on a higher level, and the states can then be the numbers. This also gives us some hope for considering the operators as those of normal number operations.

Here is the logical operation table again:

$\circ$	T	F	O	C
T	T	F	O	C
F	F	O	C	O
O	T	F	O	C
C	T	F	C	C

Here are the cases we need to consider for which symbols are for which of strings or rings:

Looking at the logic table for an operation, if the statement ring were all T,F and the operation string were all O,C then a function's operation on it would reveal T mapping to T and F mapping to F for all O,C. Therefore, nothing would happen.

If a string were all O,C then a function's operation on it if all T,F would act like $\mathbb{Z}_2$. False reversing O,C and True, maintaining them. This reverses or leaves alone all statements of the string. So, if the string is composed of O,C then the operation is string reversal at points and not on others, but is specific to T,F in the operating string. With this, we can construct any new number.

If the string were O,C and the operator T,F,O,C then the T,F sector behaves like $\mathbb{Z}_2$ producing new strings while the O,C sector on O,C would behave as composition under a new operator. We will use this version for reasons to be explained.

Most of the functioning between operators is of the nature of true, false, open and closed, when on state rings, or statements. To get a string to converge we must find a way to get the set of true and false to change into open and closed by combining with other true, false statements. We need both groups in the operators for intermediate states so that an operator with all of these may contain operations between all of them in other operators.

So, we have arrived at:

Operators from the set: $\{T, F, O, C\}$
States from the set: $\{O, C\}$.

Operators then operate by $\mathbb{Z}_2$ on statements to produce new statements and by submatrices we find them to be the property of addition and multiplication under O,C. The numbers are then in binary, and we have a correspondence of:

$$\text{Open} \equiv 1 \qquad \text{Closed} \equiv 0$$

The top right part of the logical operation table looks like addition of numbers, while the bottom right looks like multiplication. Although these are in the operator, they do not behave in the same way exactly, unless we use the right encoding. It would be interesting to use this operation also for these purposes. To make them operators of their own and for reference consider the following binary operators on the rings:

+	C	O
C	C	O
O	O	C

×	C	O
C	C	C
O	C	O

With these, we can define the normal acts of addition and multiplication between statement rings. This completes the behavior as numbers. It is interesting that we can formulate a consistent system with open and closed taking the part of binary operators on intervals of a decimal point system to obtain operations between numbers as well when we combine them.

We find that in reversing a string, false acts like open in the addition operator. This makes sense, for to get the other state of true or false, we must go through open. Also, ideally, we would like the end conclusion of openness or closure to follow the last step for which we obtain true or false. This will come in as important later.

For instance, the union has Closed as a multiplicative identity, the intersection, Open. The sequence operator exists with this for both True and Open. The union has Open as additive identity, the intersection, Closed. There is no additive identity for the $\circ$ operator between sequences and rings.

The sequences, or strings, have rational elements multiplying each irrational in the series, which we need for the multiplication identity, and for uniqueness of series in full over the reals. They operate on rings to produce new ones with the same properties. We can do ring checking with irrational numbers, and sequences are otherwise rational or irrational. The rings have their multiplicative identity in one element, a zero for the rationals. For the sequences this is identified with one string, the all open one.

When we do a comparison test, which is the structure of testing with another series accompanying a first, the result is rational or irrational, but by a good bound can give us the logical value in the larger program structure. The result of comparison is not a number in the set of statements and converges to a rational under certain provisions, such as termination. The sequences therefore constitute an equivalence class.

Statements and sequences feedback from results obtained by operations on the rings. This is the normal mode of operation. For a result to be out of sequence, it is not on the ring. Therefore, rational results terminate and may not have an interpretation back in terms of the system, which is now self terminating and closing, so long long as new sequences are not admitted.

This system of rings 'lives', on the open interval with Closed at 0 and Open at 1. These represent the states for operator truth. Using this same left right organizational scheme, we consider the other logic tables. For the operator on another operator, we find Open and False with the 1, and with the operator on the ring we get True and False at 1. In the operator on operator, it is False at zero to True at one. In this sense, distance from zero is also accumulated certainty on average, yet bears no correlation with operators on operators.

These place different emphasis on different attributes yet are quantifiable and reciprocate, being related to themselves via their group. One can be seen as one of $\{O, T, F\}$. Zero can be seen as one of $\{C, T, O, F\}$. We find the topological condition intact. True and False run off in a nongeometric perpendicular sense to the rings into the making of strings. The sequences can contain rational pieces of the rings, and these reciprocate by moving around in a loop as the program runs as we multiply by other rationals.

The irrational digits are enough to determine in between if a sequence suddenly goes over to constant O or C or T or F for a distance equal to the distance from the zero position to the beginning of the constant, then the state has not changed half the time, and we attempt to consider it is as fixed. With any resolution this is not enough for certainty, and what we get is a sort of devil's staircase when expanded in a normalized notion of time from 0 to 1.

If we reach this half way point in string length, have we saturated the bounds of its certainty? A different staircase would have a different parameter for dividing, so it might or might not be reasonable. In most cases, it is simply the time spent on this portion near 1/2 that determines the likelyhood that it is greater than %50 certain. In our case, this only holds as exactly %50 up to about two thirds of the real number line interval, and then it chaotically starts jumping up and across again. This is still however a pattern.

8 Logical Groups and Functions

With these logical groups, we can find the natural groups which generate the desired functions. Elements of group structure are permutations, operations on sequences, modules of bases, and group orders determined by auxiliary sequences. The importance of this is modular functions that have variable rates of interstitially distributed logical values. We can then describe a function and perform relaxation to a descriptive function, by way of finding a standing solution of limiting case. As a process we are attempting to find the combination of rigid and fluid that yields the given desired nature of function. This is part of a larger process, where we build base functions from natural properties of the ring. With the other functions defined by this process, of the enumerative type, we can easily take the power or limit of such a series or sequence numerically.

This is important because we can use the tools of mathematics in certain cases upon these sequences to arrive at results of their interoperation. Note as an example that the logical values operate as clarifiers in many times, the same way that these statements do. Consider the union of all random sets. This set is clearly open in the sense of the topological character of its boundary. It can only form an open boundary as each subset in a limit has an isomorphic interval with the same structure as $\mathbb{R}$. The intersection is closed, as it is clearly not open.

The repetition of a function can indicate the product of the sequence with itself has an on off nature, or one that converges to open or closed. The simplest general function test is this; to see if a sequence converges under an operation. Although, this is not a simple test. The above examples illustrate that we would like to find a general correspondence for the construction of these logical sets.

In sum, between the rules on these rings and the rules of a logical system, these are sequences of logical states with a group property, such that under multiplication or compositional intersection they have no multiplicative center, and we arrive at the same result despite a linear displacement. They are as a union, in an equivalence class, and represent a group of objects that transform similarly under operation. We hope to create an algebra out of these operations. In this paper we build from both sides to reach this goal.

These objects generate an algebraic structure with their groups. We are afforded more groups with more structure, such as the TFOC system so far, when we have more elements. Since the answer to this question is infinite, how the group determines their multiplicative result is a point of study. We eventually wish to generate polynomials to describe the system based on the group structure. Each set corresponds to a geometric transformation in a modular group, corresponding to the structure of the rational point set.

Classifying all groups would be hopeless, so we focus on the interrelationships that exist between different sequences as based on the elements of groups that transform under the operators. These operate upon groups and groups created from groups. The focus of this is on deciphering an operator's transformation rules upon an infinite collection of these groups. In this case groups of rings or sequences. With this we build towards the infinite continuum of continuous logical functions. By doing so, we focus on the emergent properties of logical systems instead of on the underlying sets of logical state. This allows us a freedom to construct arguments with a more continuous form of logic, by using these sequences as the foundational objects. These are the rules that define their interrelationships.

Uniqueness is primary, among sequences and rings, and is important in large part because it suffices generate structure. We need this to to produce rings that work with one another to produce unique rings or sequences. The results are commutative in as much as the underlying group is. To symbolize a sequence by a number can be done. It is many to one unless each sequence is unique. Also useful is a chart with the number specifications and according logical values. With each new sequence recall that order does matter, although net offset does not. This is in part a property of having so many numbers, as copies of $\mathbb{Q}$.

How these multiply is complicated, but can be carried out simply with a table. The results are new rings, and how these rings work together, is the foundation of the theory, when we work in higher dimensions. Rather than have the objects specify the relations, the relations are used to specify how these objects interrelate, and the viewpoint becomes less objective. However, the rules are not in using these numbers exclusively, but are also used to define them.

We can construct strings by using many random numbers, together in a sequence. We can also find ones randomly in convergent patterns. This constucts a new random string by way of looking at the convergence to a new string. The new string is one of $4^{\mathbb{N}}$ in total, with N the string length, each corresponding to a different configuration of symbols. We can see here that there is some variability in the choice of irrational and rational numbers, when all that matters is the sequence order. We find that there are as many sequences as there are real numbers, as one sequence alone has $\aleph_0$ entries. There are $4^{\aleph_0} = \aleph_1$ such sequences.

These as individual strings are in the number of ($\mathbb{N}$). These are actually strings of every kind, and with their sequence associated with a limiting point, we find that each state sequence behaves as a real number. Since a rational multiple of an irrational behaves differently with each rational. It is interesting that the rationals all behave the same under the irrationals. This suggests the division into two groups with different purposes. One rational point among every set in the collection is allowed. This means there is one rational group as the multiplicative identity. With this we find that since each string returns a number under composition and converges to a number as a series, it is essentially a number which behaves like a localized function.

But rather than being a function of something, it is more a variable at the same time. These functions can change the functional behavior of other functions. Inverses in this problem play a particularly important role.

It is potentially true this one rational point exists on a sequence and its inverse, but then there are two, and in potenentially different locations. It would be symmetric if they and their inverses were the same object. Then we have only one rational point among all these series. We therefore have a multiplicative identity, in an arbitrary random rational $\neq 1$. The laws, in breaking, have reemerged delocalized.

If we consider rings with arbitrary sequences it is not clear what the inverse of a number is. But if we take some examples of state rings, it becomes clear that the reverse string with symbols reversed is not the same string. For example, consider the binary expansion of $.25 = .010000$. Under interchange and reversal we get $.111101 = .953125$. It is clear they are not related by a simple inversion or subtraction relative to the period, but it may be something deeper, such as reversal with respect to a circle in space. The numbers as rings, do not have a clear inverse but the operators do.

Since the identity is the rational set, the conclusion is that for a number ring to have a reversed string, and true false interchange, or in other words, an inverse, would require there exist an element in multiplication with it producing an element of the identity. This is the radical to rational number operation which is claimed to be unique, given a prime factorization of radicals yielding unique irrationality as in a prime radical. From this, we can deduce their behavior is like that of prime ideals. Given this is true, the inverse should exist for the numbers, but only as the ring itself. It becomes, it's own inverse. Not every one of these numbers has this property of reachability, and thus not all of them have inverses.

9 Series

Part of this arisese out of an effort to understand what happens to sequences of rationals and irrationals as they converge to a number. Are some numbers unreachable? Can a nonzero number be reached?

An example of a valid series is:

$$\{\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{5}}, \frac{1}{\sqrt{7}}, \frac{1}{\sqrt{11}}, \frac{1}{\sqrt{13}}, ...\} \tag{76}$$

Another is:

$$\{\frac{1}{2\sqrt{2}}, \frac{1}{3\sqrt{3}}, \frac{1}{5\sqrt{5}}, \frac{1}{7\sqrt{7}}, \frac{1}{11\sqrt{11}}, \frac{1}{13\sqrt{13}}, ...\} \tag{77}$$

These of course converge to zero. With:

$$P^* = \{p_n = P_l^{o_l}...P_m^{o_m} : \exists o_j : o_j = 1, N_{o_j} = 1 : \forall n \in \mathbb{N} : P_n \in P^*\} \tag{78}$$

These sequences can be written:

$$\{\frac{1}{\sqrt{a}} : a \in P^*\} \qquad \{\frac{1}{a\sqrt{a}} : a \in P^*\} \tag{79}$$

The list of all prime radicals, and the list of all prime divisor rationals times the list of all prime radicals. As an interesting example we could also analyze a list of all numbers containing an odd power of a prime factor as radical divisors for this list.

$$P_{f,o} = \{p_n = P_l^{o_l}...P_m^{o_m} : \exists k : o_j = 2k + 1 : \forall n \in \mathbb{N} : P_n \in P^*\} \tag{80}$$

Then:

$$\{\frac{1}{\sqrt{a}} : a \in P_{f,o}\} \tag{81}$$

Because the odd power of a prime factor and the square root add to an integer plus one half power, this list only consists of irrationals. The outside or inside list can of course be different when they each satisfy our conditions, or when both are prime:

$$\{\frac{1}{a\sqrt{b}} : a \in P^*, b \in P_{f,o}\} \tag{82}$$

$$\{\frac{1}{a\sqrt{b}} : a \in P_{f,o}, b \in P^*\} \tag{83}$$

Yet we cannot have both under the radical, since $\exists o_j = o_k : P_j = P_k$ that is the same in both expressions.

$$\{\frac{1}{\sqrt{ab}} : a \in P^*, b \in P_{f,o}\} \tag{84}$$

Since both contain the same primes in an overlapping manner, this sequence would not be a valid element in general unless we take $P_n \geq 7$. If we translate into a function as the constructor for the series then the summability of the series means the function is integrable. This implies something interesting, if we carry this one step further. The function can be convolved with others. For example if we take two sequences:

$$\{\frac{1}{\sqrt{b}} : b \in P^*\} \qquad \{\frac{1}{\sqrt{b}} : b \in P_{f,o}\} \tag{85}$$

And use them as two separate statements. By multiplying these rings from 7 onwards, we can obtain information about these series. Since we know they converge to a rational, it is clear that their product does as well. Each is an element of our ring, and we obtain information about numbers with prime factors that include single or more odd powered exponents in relation to the primes themselves.

Since a sequence or a ring may converge to zero such as this, such cases are less interesting at answering a question between two irrational series as a finite number would be. The limit of one is absent in the ring of operations and statements. Because the multiplication of two numbers between zero and one results in another in this interval this is indicative only of a fixed group of real numbers, the open interval. With an open interval the limits to the boundary although capable of being defined, are extendable in this limit as the set size increases by open sets.

What kinds of questions can be answered by properties of these series?

Can we distinguish a fundamental irrational part multiplied by a rational prefactor?

Can we determine if a number is composite or prime?

Can we take the limit of two series simultaneously?

We wish to create a set with numbers that have prime-like properties, in that they exclude numbers with even powers of primes. These map back to rationality.

We look at:

$$P^{\circ} = \{p_n = P_l^{o_l} ... P_r^{o_r} :!\exists o_j : o_j(1 + 1/m) = k : \forall k, n \in \mathbb{N} : P_n \in P_n^*\} \tag{86}$$

And the difference between:

$$A_m = \{a_n = \frac{1}{P_n^{1+1/m}} : n \in \mathbb{N} : P_n \in P_n^*\} \tag{87}$$

And the use of no factorizations leaving rationals:

$$B_m = \{b_n = \frac{1}{P_n^{1+1/m}} : n \in \mathbb{N} : P_n \in P_n^{\circ}\} \tag{88}$$

These sets should be related, for prime factorization is fundamentally related to factorization in general, and the second set is based upon the first. We can then, compare convergence of their irrational decimal expansions. What concerns us, is if the limit goes out of the domain: $[0, 1)$. These both converge to zero so they are ok. These help answer our uniqueness condition, unless there is none at zero.

One should note that the individual irrational numbers in these series are actually numbers congruent to the rationals. Our series of actual numbers are seen to be convergent to rationals or irrationals, our operators cannot in general be depicted as numbers, and our statements can be seen to be equivalent to irrationals. Although we could construct a statement of statements, clearly, this would require numbers in excess of the reals, $\mathbb{R}$.

We can use the following (recursive) definition for the set of primes:

$$\mathbb{P} = \{r : \frac{r}{q} \notin \mathbb{P} : \forall q \in \mathbb{P}\} \tag{89}$$

Or with the natural numbers:

$$\mathbb{P} = \{r : \frac{r}{q} \notin \mathbb{N} : \forall q \in \mathbb{N}\} \tag{90}$$

This recursive property is shared. One can see in this definition that the primes are not like the rationals but are more irrationals, in that each new member is exclusively non-divisible by the previous ones. This is analogous to the irrationals, because there, we have a missing group, the rationals, that if two numbers are divisible to a rational, then they are rationally related. If we restrict our members to being irrationally related irrationals, then we have sets whose behavior is not unlike primes.

There exist other interesting functions. Consider for instance:

$$\frac{1}{a\sqrt{f(n)}} \tag{91}$$

As we identify:

$$f \leftrightarrow f(n) \tag{92}$$

$$F = \lim_{n\to\infty} a\sqrt{f(n)} = f(n) \tag{93}$$

Now, a converges to:

$$a \to \sqrt{f(n)} \qquad F \to f(n) \tag{94}$$

If $f(n)$ is on $\mathbb{R}\backslash\mathbb{Q}$ plus $\mathbb{Q}$ then this is equivalent to $\mathbb{R}$, and $\exists b = \frac{1}{f(n)} : b \in \mathbb{Q}$. Then, both converge and both were irrational. They must have the same sequence and therefore be equal. This holds for the whole real numbers and only at values $f(n)$ specifies. This holds if $\sqrt{p} \in \mathbb{R}\backslash\mathbb{Q}$ iff $p \in \frac{Q}{P} : Q, P \in \mathbb{P}$. We can also define functions strictly on $\mathbb{R}\backslash\mathbb{Q}$, which is valid, but leaves out the rationals. Now on $\mathbb{R}\backslash\mathbb{Q}$ we define a limit that behaves like the contained function:

$$F(n) = \lim_{n\to\infty} a_{op}\sqrt{f(n)} \to a_{op} = \sqrt{f(n)} \tag{95}$$

The limit with a_{op} behaves and acts like a $f(n)$ from $\sqrt{f(n)}$ among rings, through this limit. It acts like a squaring operator. We can with group rules determine how neighboring ones behave. Then, based on rules, we can find a function behaving like any of a large variety, when we consider coupling the functions to each other.

$$\frac{1}{a\sqrt{(b-f)(c-f)(d-f)}} \tag{96}$$

A function of this variety is a polynomial in f. This function, must have at least one unique root. No multiplicity in all roots guarantees that they do not have (multiplicative) rationals of the form of a square for instance. These numbers simply factor under a radical. Thus, we need at least one non-squared prime factor in the denominator or numerator with a rational, to generate an irrational number. To have no relation to the previous ones we find we need the primes.

Other irrationals are ok, so long as they are not irrational roots that can combine to overlap to make a complete rational. In other words, unique. If fractions are always in reduced form and we use only primes, this is provable: $\sqrt{p} \in \mathbb{R}\backslash\mathbb{Q}$. Thus our functions take on all of: $\mathbb{R} : [0, 1)$. The irrationals give a prescription for finding a sequence (infinitely non-repeating) for which the order of elements is all that matters. This is only the circular order that exists with unidirectionality. As a whole, these numbers behave like primes.

This is important for uniqueness with the product leading to a rational if and only if their irrational digits match. They also match when they are a rational multiple. This relaxes our constraint on the numbers if we want to be able to generate these numbers to carry rational information. We would have duplicates in our list of irrationals if any irrational were allowed in our list. These numbers have uniqueness in number coming from the rings, and this property is important. The ideal, and non mutual overlapping irrational set is generated by the primes. This insures that we do not get a basis with repeats or rationally related irrationals.

It holds $\sqrt{p} \in \mathbb{R}\backslash\mathbb{Q}$ if $p \in \frac{Q}{P} : Q\&P \in P^*$. At this level, we include all fractions of one prime over another. By doing this we generate a maximal set of numbers for which they are all mutually irrational. These numbers serve as a basis, for which prime checking is done via multiplication, yet they do not give a formula for the primes. These represent the series elements. If we arrange things correctly we can find a limit function that is the conjugate half of the number and therefore behaves as convergent to a function. This is interesting because it expands our notion of what is possible with these sequences.

Numbers actually need not be prime. They may be coprime and possess at least one odd power for a factor. This will satisfy reduced form as a radical that is not rationally related to another in the set. But this must hold for all the numbers mutually, and so in the limit of set size going to infinity, we must use primes as our set.

We use the sets:

$$\alpha = \{r_n = \frac{A}{B} : n \in \mathbb{N} : A < B \in P_*\} \tag{97}$$

$$\beta = \{r_n = \sqrt{\frac{P}{Q}} : n \in \mathbb{N} : P < Q \in P_*\} \tag{98}$$

To form:

$$\Omega = \{r_n = a_l \cdot b_m : n, l, m \in \mathbb{N} : a_l \in \alpha, b_m \in \beta\} \tag{99}$$

An alternative process for forming a set of primes without division is the following set theoretic composition:

$$C_1 = B_1 = A_1 = \{1, 2\} \tag{100}$$

Then we form recursively:

$$A_n = A_{n-1} + \{n\} \in \mathbb{N} \tag{101}$$

$$C_n = A_n/(A_n \cap (A_{n-1} \times A_{n-1})) \tag{102}$$

$$B_n = B_{n-1} + \inf(C_n/B_n) \tag{103}$$

This produces the list of primes:

$$B_\infty \equiv \mathbb{P} \tag{104}$$

This is a way of saying that all numbers that are the product of numbers are nonprime, and those produced not from products are prime. With this we can produce primes by exclusion. This is a unique algorithm without the use of division, and only multiplication. It is reiterative unlike a seive, but is like one in that we generate all products of all primes in the previous list of primes and add a natural number to supply it with natural numbers. If we find that n is already included in a list of products then it is nonprime.

10 Logical System

We can translate our logical system into a topological number system. We accomplish this by making the following mapping:

C: Closed Set
O: Open Set
T: In Set
F: Outside Set

This is by far one of the simplest way to translate the rules of this system into a topological theory. The goal of this is to find an isomorphism between the structure of irrationals and a rule set. With open and closed we can express relations between numbers as relational, opening up our description of number. We will instead use a different dictionary.

A set relation is specified by closed, with direction assumed by the direction in which a string is evaluated. Periodic gaps between closed statements indicate other sets that are contained within this set or sets that contain this one with true, false, and open as states. Sets contain the sets ahead of themselves, and are contained in those behind.

Another useful representation is:

C: Boundary
O: 'Set Unit'
T: True, unidirectional
F: False, unidirectional

This is useful for the definition of the sequences whose truth value is determined by evaluation as a whole, and not so much for the strings we will develop. However, the set unit is a good concept, for it can be used to indicate sets for which there exists an open relationship in one direction as we move along a string. These sets being skipped by the logical evaluation allows us a degree of topological transitivity.

For these strings, we will find the truth of a string nontrivial. If a string is left right true false symmetric, then it is valid. Given that these are relationships about sets contained in one another, they must be symmetric relationships to be universally true and hold for all elements.

With uneven strings (non-fixed set size) we can get interesting behavior. There exist set relation sequences for which their infinite number admits metastable strings with functional properties that nevertheless evolve with step number. Functions that change the functional property of others. Since sets within sets of this number ($\aleph_1$) are a part of the rules of the sequence, there are a spectrum of programs that are equal in number to $\aleph_1$ in countability in the number of sets and therefore may not be prone to the same laws as objects such as Godel strings.

These automorphic strings create a cyclic process whereby true and false simultaneously indicate closure relations, sequences or orders of operations in a potentially infinite heirarchy. These operations open and close channels or add and remove boundaries, rearrange the space, and reorient the evaluation of the string, by reorganizing the set relations to new true and false relations. A beginning is chosen by an intersection over a union of rules.

The hypothesis is that there exist consistent infinite strings with no blank connections and no unused spaces with these properties known as free strings. These may not exist because of Godels theorem, however, they have an admitted possibility of existence, because they do not question provability, and are fully in $\aleph_1$.

Within each set is the pattern, of the way and rules by which to consider all other sets. The order of operations is determined by a method of greatest constraint.

We explore all relevant levels (which can be all) to reach a beginning node, or set of equivalent beginnings, and then take the first step. We re-evaluate the entire object to determine where to begin again. This can make for a very chaotic playout of rules, and allows for complex reiterative behavior.

Consider the following string:

$$...TCT... \tag{105}$$

This is a single empty set connected twice over to itself. Now consider the following string:

$$...TCTFCF... \tag{106}$$

This is two sets, each containing the other, with one true and the other false. Since we have the reverse direction to work with as well they say more. The first set to the left, claims the set at the right is true, and that itself is false. It also claims its relation as a set item to the set at right is true and to itself, is false. The second set to the right claims the set at the left is false, and itself is true. It also indicates it is a false element to the other set, and true to itself. With this we can see that we have going left to right, the set at right as true within the set at left, and itself containing this set as false. We cannot support this conclusion, and hence this string and this situation is a paradox, and a rather involved one. One can see if one draws these sets and how they connect, that a paradox is essentially a logical Klein bottle with true and false on the same surface.

One question is if a statement that declares these two false can disentangle a paradox. Consider the following string, where a paradox is 'contained' in a larger statement that proclaims it as false. In this way we can see how a set that contains another can perform an operation so as to disentangle a paradox, and reach a refined set of sets with minimal changes in the string.

$$...FFCFFFCFTCT... \tag{107}$$

Since both having False applied to them is one consistent outcome, and the others are not, this operation resolves to application of the new set to the two previous. Using the operator table, we find that false and false go to open instead of true in this non true/false exclusively constrained system. This string becomes:

$$...FFCFFOCOFCF... \tag{108}$$

Since Open is open to either state, this is the final string. Now, set one, our set that declares the relationship of the other two false, contains these as open and false, and is contained as open and false by two elements each in the other. This evaluation reduces our string by taking False to open in the contained false set. When the second two flip and false goes to open, the original set does not change. If it simultaneously changes, we get another paradox.

It takes a greater number of steps to reach an end state consistent string with a greater number of initial group relationships, and these end states are not always reached. When they are, we know the original system is solvable. As with many systems there may be more than one way to reach a correct answer.

There are again two sets of logical values for a set. With the direction of evaluation, the values behind indicate the set possesses a given property to those sets that contain it as an element, in reverse order. The values ahead indicate: the sets it contains as elements possess this property from the evaluation, in normal order (along with the direction of evalutation). There are two complementary ways to evaluate; left to right and right to left.

With this, we can see there are two countable infinities for sequences. One is the set nesting level, and the other is the sequence length. The sequence is now a list of their somewhat transverse relationship; one where we look at the sequences created by sets within sets. This is seen in both directions, from the very small interior set, to the very large exterior set. These sets are contained in each other, and hence, we are really working with an isotropic set, one without inside or outside, and fully hypothetical.

Since for now we assume every $T \leftrightarrow F$ is a bisymmetrically true / false relation, these are identified with sets. They need application as a relation.

We can assume that the elements between closed apply to groups and closed we may leave as solitary. We can get a relation of the set to the sets so contained in a set. This relation is a hypothetical union and intersection in some combination, between all sets. Since this is the result of the union or intersection, the implication of this reversed implicates the ones that in the union or intersection produce this result. Since they are contained in each other it is in general the intersection. This is essentially a function as implied by a relation. They are in this sense the two inverse elements under the intersection. We do not know which precisely applies however as this operation is not unique under the inverse.

An Open indicates exclusion of sets from a set inclusion when we begin at a set boundary. The sets relation under intersection is the sequence to the front of Closed. These can contain closed ends or whole sets. Only with whole sets can we have a complete union or intersection. The boundary is irregular as these are based on irrational numbers, so, instead, we cannot guarantee we have the same point referred to with a mutually closed boundary, and this is excluded from becoming open when we use multiple sets to find the digits of an irrational.

We assume from this a few things: Open is equivalent to a set connection which leaves the set items intact. Thus open is equivalent in being exclusion when we use this symbol to skip groups, as these items are on the same level and outside of consideration for determining the true and false relations that exist between the other elements with definite truth values. Although, open is flexible, and may change type. If we get X as a state from for example two directions, and these each are implicative of a mutual connection, we can often get X.

Now, lets look at an example that has mathematical relevance. This is the Fibbonacci sequence:

$$1, 1, 2, 3, 5, 8, 13, 21... \tag{109}$$

The recurrence relation is:

$$F_{n+1} = F_n + F_{n-1} \tag{110}$$

$$\{F_1, F_0\} = \{1, 1\} \tag{111}$$

With the set operation of addition we can form this sum easily, one number for each set's number of contained set':

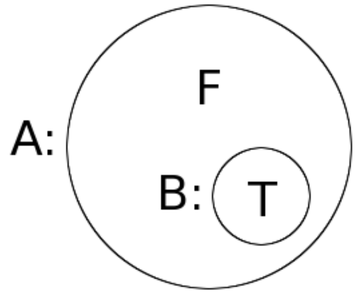
$$...CTTCTTCTTCTTCTTCTCC \tag{112}$$

Results in the sequence of set numbers as:

$$...21, 13, 8, 5, 3, 2, 1, 1 \tag{113}$$

This repeating pattern of: CTT is enough to keep the sequence going, after an initial CCT in reading right to left. We see the first indications that as the number of sets increase the space for program type code left over becomes smaller. This is with a fixed length sequence. With an infinite sequence we get the behavior of our reals. As a finite one becomes smaller in this context it becomes a sequence. There exists a spectrum of behaviors of these objects from a string to a ring. The spectrum goes around in a circle to reconnect to itself, in that these are the same objects, only in different guises.

Hypothesis: there is a way to write a seqeunce for the primes. Note that this is one of a non recurrent nature except in its nature of asking a question of divisibility repeatedly on the natural numbers. In general, our logical system reduces to the production of logical functions. With the following diagram notice a logical statement exists by the set relation mapping True to False as we move outwards.



Going inwards, false in this logical function gives true, and going outwards, true in this logical function gives false. This logical function is taken as locally bisymmetric in this manner. The logical function is defined in both directions by this, and actually exists with at most $(3N)^N$ relationships. This is but one among many parallel relationships. A statement is implied:

$$T = \phi(F) \qquad F = \phi^{-1}(T) \tag{114}$$

These ϕ^{-1} are determined by the logical comparisons. How do we find in general the ϕ when only given T and F? i.e. there could be multiple intermediate operations in general. Our question is: how, with the inverse union and intersection, do we determine uniquely (and is it possible to) the set of:

$$\infty ... \phi_{AB}\phi_{BC}\phi_{CD}\phi_{DE}\phi_{EF}\phi_{FG} ... \infty \tag{115}$$

This is the logical function, contained in these interrelationships. When we take the limit of the number of statements to infinity it becomes continuous.

These statements or sequences are infinite in extent, and overlap like fibers running in both directions to meet at infinity. They also come in an infinity of different parallel statements with each infinite in extent, overlapping along their length. They are not limited to one dimension when we have them contained in each other. As a graph these are free in three dimensions, as any to any relationships. Recursion can be employed at any level. Question: Can we form a paradox in one direction in this string system?

As a comparison, consider the following values:

$$T : \forall \in A = \forall \in B \tag{116}$$

$$F : \forall \in A \neq \forall \in B \tag{117}$$

$$O : \exists \in A = \exists \in B \tag{118}$$

These are also compatible with our set notions. Our logical functions come out in interesting ways. For instance, the long chain above is just one of many. The number of sets is large, and varies depending on the structure of the sequence. The most interesting thing about these sequences is they are left right symmetric with various periodicities, with a logically consistent condition on their truth values. This implies that set A is symmetric with set B at the $(n, N-n+1)$'th (from each) unit with respect to inversion of which contains which. We arrive at this by going from one set to the next or previous and meeting. We find elements must agree between the two since they are single symbols.

The following is the condition as we go from a position at center (A) to one at the right (B):

$$\phi_n^m = \phi_{N-n+1}^{m-1} \tag{119}$$

$$\phi_{N-n+1}^m = \phi_n^{m-1} \tag{120}$$

n: indicates the sequence repeats every n sets with N the set length. These $(n, N-n+1)$ are the distances to the points where the function must find agreement with the previous set along the sequence. Lets say we have 8 sequences:

$$\phi = \{\phi_1^1, \phi_1^2, \phi_1^3, \phi_1^4, \phi_1^5, \phi_1^6, \phi_1^7, \phi_1^8\} \tag{121}$$

With only eight, we would find ϕ^1 dual to ϕ^8, ϕ^2 dual to ϕ^7, ϕ^3 dual to ϕ^6, and ϕ^4 dual to ϕ^5. Each has an offset by one, and they form a net on the periods of the ring. Up to half the period the sequences come out dual to those with a period the interval minus their period, when we evaluate in the opposite direction.

If we divide into odd and even sets then this construction separates into mirror images. Now, M equals the image and M^{-1} equals the mirror image. These read the same one direction as the other. To apply this understanding the mutual relationship of which is contained and which contains is inside/outside symmetric locally and of a symmetric local logical value. If the sequences are validly defined on even and odd sets, it does not matter what these sequences express. The sequences are inside outside symmetric when $M = M^{-1}$.

A topological question is sometimes devoid of preference in representation to what is contained, and what is the container. Also, the given that logic be symmetric with respect to time, admit a structure where reading right to left and left to right should preserve the logical meaning of the statement. This system so far preserves this aspect of logical structure. We have an identity when: $A \subset B$ and $B \subset A$ so that $A = B$. This is the symmetric interpretation of the string, and as an indication of having an identity these contain each other. This symmetry allows us to use symmetric implications in places, or to guess what the logical operations were that lead from one relation to another. We can use this symmetry to model a logical function, of arbitrary length and complexity. Sometimes these are ones of infinite length, when we use other recursive elements. A question, in the language of logic.

Ahead, every group indicated is a group contained in the previous, with a logical value so indicated by the group with the logical values immediately succeeding it. Behind, every group is a group that contains this group, with a logical value within this group given in reverse order to the evaluation. In this, a group in the center is an element of the sets that come before it, and after it are the sets that are within it.

Sequences and rings that go both directions are symmetric sequences or rings. If we were to take every nth item of the set we would find them covered sporadically by the actual groups of the permutations of the sets. In other words if we take the nth group, and then the nth group again, and repeat, we don't cover all N groups until we have gone around at most N times. In some cases we do not cover all.

We do however have a symmetry. The relevant numbers are the period n, and how they line up with the period $N - n + 1$. The plus one is from counting our start and end. Our question is, which lengthed strings and for what ratios of N or $N - n + 1$ to n do we find our rules satisfied? As well, what is the base function generated by agreement between these? The resultant strings are left right $n/(N - n + 1)$ period symmetric. These are the encloses/enclosed inverses. They have as well two different periods in two different directions as the same sequence. The same sequence has two periods specified by the reiterative numbers for n and $N - n + 1$ to get back to where it's starting point is. These are our two periods:

Left: Period: n and covering number: $\frac{LCM(n,N)}{n}$
Right: Period: $N - n + 1$ and covering number: $\frac{LCM(N-n+1,N)}{N-n+1}$

These must satisfy the rules for ϕ to agree and have commensurability. How do these functions change as n and N do? Irrational numbers, which we can model our functions off of have many periods.

Given the previous, the two patterns match up, although out of order, when:

$$\frac{LCM(n,N)}{n} = \frac{LCM(N-n+1,N)}{N-n+1} \tag{122}$$

As an example, consider $N = 5$. For $n = 1, 5$ they are unequal. For $n = 2, 4$ and $n = 3, 3$ they are equal. We find they are equal when:

1.) N is odd
2.) n and $N - n + 1$ are coprime to N
3.) when N is prime, all n other than N and 1 function

This only works therefore when the two lengths that add up to the total, tile this total in such a way they both cover equal numbers of sets. This is when their numbers are coprime each in pairs, to the total number. Their sequences come out in unique ways, but are individually ordered. These are also capable of indicating a sequence of operations.

Because of the symmetry requirements, the interval is symmetric about the midpoint. In between, sequences must be symmetric about $N/2$ such as in the following:

$$...CTFOTFFFTOFTC... \tag{123}$$

These are the points that must agree, going to the left or right of $N/2$, however, order does matter. The ordering depends on the ordering of the multiples in a pattern relative to the other patterns in an N sized modulo space. This indicates that the sequences are different, depending on the size of the space and the number of sets, and because the ordering of the digits in relation to the string is unique with each pattern.

These can have very different structures. One which goes every two, in a space of nine, and one which goes every eight in the same space, match up at their common multiples. These are 9,8,7,6,5,4,3,2,1 for eight, and 2,4,6,8,1,3,5,7,9 for two. One can see the order is backwards on the set of sets in the first case, and we have evens and odds in the second, in ascending order. These represent transformations of the set into different orderings, and when they change they have interesting results. The counting begins from one of two closed set relations, or two boundaries, in from each side. Only those sequences that have the same period of total coverage in n and $N-n+1$ have agreement.

For another example we look at those patterns for five elements. The following is a depiction of the sequence of sequences landed on, as we rotationally cover the interval with sequences.

	1	2	3	4	5
1	1	2	3	4	5
2	2	4	1	3	5
3	3	1	4	2	5
4	4	3	2	1	5
5	5	5	5	5	5

This indicates 4 and 1 and 2 and 3 should agree. The above gives their relative ordering. These numbers are of the whole sequence of sequences and indicate groups of spaces covered. These further match up, but out of order, and generate sequences from their mapping. This is so long as we associate 1 and 1, 2 with 2 and so on. By doing this we satisfy the group rules for their association. This establishes a mapping. We can use this to understand what the sequences of the functions are in terms of their natural mapping from these modulo sequences.

Since these are the only ones which match up, the fives are out. As in general are the N. We get one of these tables for each N. Sometimes, as with nine, most do not match up and some do. The ones which match up are 2 and 8, and 3 and 7. The other numbers are non mutually prime, and therefore, ones for which they become commensurate at some number of spaces, but not in a manner such that they cover all spaces.

The conditions above ensure incidentally that in all cases where we have a matching condition between the lowest common multiples over their periods, we get a number that fully covers the set. Their coprime nature does not hold mutually, but does hold for each as compared to the total interval. They may, or may not be coprime to each other.

The ramifications for a completely coprime set imply total covering of size N for all bases. This is however not possible with all natural numbers as all natural numbers are not all mutually coprime. We are struck with a case of two parts to a sequence, its 'agreement' conditions, which come at periods n and $N - n + 1$ and correspond to numbers coprime to N, and another part; those that have no agreement condition.

These two parts, bear a striking resemblence to the parts of our numbers representative of sequences, the rational or the periodic, and the irrational or those without agreement. These without agreement still have a mapping, as a potentially smaller set of potential logic values.

If we choose to interpret this system as one not of equations but instead, as what a sequence represents (i.e. a permutation of 3), then we move a step closer to understanding the true meaning behind these numbers.

11 Group Structure of the Irrationals

A faster process exists only if we can use the extra information of $r = \sqrt{pq}$ based upon it being a square rooted two factored product of primes.

Let:

$$\alpha_P = \{r_n = \frac{1}{\sqrt{P}} : n \in \mathbb{N} : P \in P^*\} \tag{124}$$

Be the list of all prime inverse radicals. Then take all products as: $\alpha_P \times \alpha_P$. Then multiply by the unknown irrational, or inverse root of a product of primes or number. This seive will produce all factors without powers as rationals for all the prime factors a number has. This is merely a straightforward seive.

If we define:

$$\Omega_P = \{r_n = \frac{A}{B}\frac{\sqrt{P}}{\sqrt{Q}} : n \in \mathbb{N} : \frac{A}{B} \in \mathbb{Q} : P, Q \in P^*\} \tag{125}$$

This is the coset of the irrational prime radicals under the rationals.

Then, our question becomes: is $\alpha_P r \in \Omega_P$?

There is a unique answer based on the given information alone. Yet it seems non-reversible. We find each radical produces with the composite a rational multiple of the other. This means if we know one, then we know the other.

Letting:

$$\{\alpha = \frac{1}{\sqrt{P}}, \beta = \frac{1}{\sqrt{Q}}, r = \frac{1}{\sqrt{PQ}}\} \tag{126}$$

Then:

$$\alpha\beta \to r \qquad r\alpha \to \mathbb{Q}[\beta] \qquad r\beta \to \mathbb{Q}[\alpha] \tag{127}$$

We produce:

$$..., \alpha r^5, \beta r^4, \alpha r^3, \beta r^2, \alpha r \in \mathbb{Q}[\beta] \tag{128}$$

$$..., \beta r^5, \alpha r^4, \beta r^3, \alpha r^2, \beta r \in \mathbb{Q}[\alpha] \tag{129}$$

We find that these must be related numbers.

If it were strictly linear this would not be certain. We need the relationship between the groups:

$$\mathbb{Q}[\alpha], \mathbb{Q}[\beta], \mathbb{Q}[r] \tag{130}$$

Then:

$$\mathbb{Q}[r\alpha] \subset \mathbb{Q}[r\beta] \tag{131}$$

$$\mathbb{Q}[r\beta] \subset \mathbb{Q}[r\alpha] \tag{132}$$

Thus:

$$\mathbb{Q}[r\alpha] = \mathbb{Q}[r\beta] \tag{133}$$

Thus, these are the same group of numbers, and we should find that either is a member of the other's when taken together. These are in this sense each contained in the other. We are afforded this as:

$$r = \alpha\beta \tag{134}$$

$$\alpha, \beta, r \in \mathbb{R}\backslash\mathbb{Q} \tag{135}$$

When we multiply by an irrational in the set of composite irrational factors we arrive at a rational times an irrational. The numbers line up to the other irrational for the rest of the composite structure. We get this behavior because multiplicatively the rationals are the multiplicative identity mod even powers. These three form a group of irrationals that transform into each other under multiplication, when r is a product. The three irrational groups from multiplication are equivalent:

$$\mathbb{Q}[r, \alpha] = \mathbb{Q}[\alpha, \beta] = \mathbb{Q}[r, \beta] \tag{136}$$

For now it is sufficient to explain the behavior as two producing the other. But what we find is that they possess a multiplicative and division group symmetry. These numbers are also their own inverses.

This makes sense given our mirrored rational construction to make them independent of multiplicative offset. The equations can also be recognized as:

$$\mathbb{Q}[\alpha\beta, \alpha] = \mathbb{Q}[\alpha, \beta] = \mathbb{Q}[\alpha\beta, \beta] \tag{137}$$

It would not have β if it did not have $\alpha\beta$ and α. There is no two fold symmetric group, but there is a one. These three groups are equivalent, but one comes from the product of two of the others. This makes sense.

As another example, what happens when we have three symmetrically? Then:

$$\mathbb{Q}[\gamma,\alpha] = \mathbb{Q}[\alpha,\beta] = \mathbb{Q}[\gamma,\beta] \tag{138}$$

And we find the elements transform rotationally. This is a group, and the last is as well. For now, we have a group table, which results in the classification of the group law as that of a permutation:

	$\mathbb{Q}[\alpha]$	$\mathbb{Q}[\beta]$	$\mathbb{Q}[\gamma]$
$\mathbb{Q}[\alpha]$	$\mathbb{Q}[\alpha]$	$\mathbb{Q}[\gamma]$	$\mathbb{Q}[\beta]$
$\mathbb{Q}[\beta]$	$\mathbb{Q}[\gamma]$	$\mathbb{Q}[\beta]$	$\mathbb{Q}[\alpha]$
$\mathbb{Q}[\gamma]$	$\mathbb{Q}[\beta]$	$\mathbb{Q}[\alpha]$	$\mathbb{Q}[\gamma]$

These are the permutation groups, which illustrate the irrationals behave like the roots of a polynomial. Is there a rule that allows us to obtain the two α and β with only γ? Since we have for instance the product of two irrationals for a number we wish to factor, and we multiply by a given irrational, and then test against other irrationals. We can find both the rational part in the first expression and the irrational part, given that we know which irrationals we tested against, and what the output rational is. This however requires two multiplications, because without this, we do not know from the previous rational times an irrational when we have the correct first irrational.

One can see that this requires multiple irrationals to determine the composites, and it requires equal in number multiplications to the number of factors. These only produce the factors, and not the degrees, but they are well suited to this task. It does however require a more sophisticated algorithm. One based around fractional powers of all inverse primes.

The results of:

$$\{\alpha = \frac{1}{\sqrt{P}}, \beta = \frac{1}{\sqrt{Q}}, \gamma = \frac{1}{\sqrt{PQ}}\} \tag{139}$$

Are under multiplication:

	α	β	γ
α	$\frac{1}{P} \in \mathbb{Q}$	$\gamma \in \mathbb{Q}[\gamma]$	$\frac{1}{P\sqrt{Q}} \in \mathbb{Q}[\beta]$
β	$\gamma \in \mathbb{Q}[\gamma]$	$\frac{1}{Q} \in \mathbb{Q}$	$\frac{1}{Q\sqrt{P}} \in \mathbb{Q}[\alpha]$
γ	$\frac{1}{P\sqrt{Q}} \in \mathbb{Q}[\beta]$	$\frac{1}{Q\sqrt{P}} \in \mathbb{Q}[\alpha]$	$\frac{1}{PQ} \in \mathbb{Q}$

Under multiplication of elements some of the symmetries are similar to that of the Klein four-group table which for α, β, and γ would be:

	$\mathbb{Q}[\alpha]$	$\mathbb{Q}[\beta]$	$\mathbb{Q}[\gamma]$	$\mathbb{Q}$
$\mathbb{Q}[\alpha]$	$\mathbb{Q}$	$\mathbb{Q}[\gamma]$	$\mathbb{Q}[\beta]$	$\mathbb{Q}[\alpha]$
$\mathbb{Q}[\beta]$	$\mathbb{Q}[\gamma]$	$\mathbb{Q}$	$\mathbb{Q}[\alpha]$	$\mathbb{Q}[\beta]$
$\mathbb{Q}[\gamma]$	$\mathbb{Q}[\beta]$	$\mathbb{Q}[\alpha]$	$\mathbb{Q}$	$\mathbb{Q}[\gamma]$
$\mathbb{Q}$	$\mathbb{Q}[\alpha]$	$\mathbb{Q}[\beta]$	$\mathbb{Q}[\gamma]$	$\mathbb{Q}$

As well, one can see that four of the irrational products occur in the cubic, as a total of five because one is duplicated. This leaves only the coefficients α and β added.

Consider the full set of permuted rational numbers as radicals whether they be less than or greater than one:

$$\{\sqrt{\frac{P}{Q}}, \sqrt{\frac{Q}{P}}, \frac{1}{\sqrt{P}}, \frac{1}{\sqrt{Q}}, \frac{1}{\sqrt{PQ}}, \sqrt{PQ}, \sqrt{P}, \sqrt{Q}\} \tag{140}$$

These form a group, because each product maps to inside the set, although they produce the product list:

$$\{\frac{P}{Q}, \frac{Q}{P}, \frac{1}{Q}, \frac{1}{P}, P, Q, PQ\} \tag{141}$$

This also forms a group, with none mapped to outside the set. Our algebraic response is that the irrationals mirror over each other like the solutions of a polynomial. A better set to take, is all irrationals between zero and one:

$$\{\sqrt{\frac{P}{Q}}, \frac{1}{\sqrt{P}}, \frac{1}{\sqrt{Q}}, \frac{1}{\sqrt{PQ}}\} \tag{142}$$

We will find this set for three primes as having similar properties to that of the prime factorization set:

$$\{\frac{1}{\sqrt{PQ}}, \frac{1}{\sqrt{QR}}, \frac{1}{\sqrt{RP}}\} \tag{143}$$

Any one multiplied by the other two results in a multiple of the other.

We can also consider the cubic:

$$f(x) = (x - \frac{1}{\sqrt{P}})(x - \frac{1}{\sqrt{Q}})(x - \frac{1}{\sqrt{PQ}}) \tag{144}$$

Rationals as products, are the 'multiples' of an irrational. There are an infinity of these. It would be interesting to come up with the analgous geometric construction that leads to and from these numbers.

Considering our polynomial in expanded form:

$$f(x) = x^3 - x^2(\frac{1}{\sqrt{P}} + \frac{1}{\sqrt{Q}} + \frac{1}{\sqrt{PQ}}) + x(\frac{1}{\sqrt{PQ}} + \frac{1}{P\sqrt{Q}} + \frac{1}{Q\sqrt{P}}) - \frac{1}{PQ} \tag{145}$$

Looking at the quadratic we have:

$$f(x) = (x - \frac{1}{\sqrt{P}})(x - \frac{1}{\sqrt{Q}}) \tag{146}$$

$$f(x) = x^2 - x(\frac{1}{\sqrt{P}} + \frac{1}{\sqrt{Q}}) - \frac{1}{\sqrt{PQ}} \tag{147}$$

This quadratic also possesses three of the same terms as the cubic. Rewriting the polynomials we obtain:

$$f(x) = x^3 - x^2(\alpha + \beta + \gamma) + x(\gamma + \beta\gamma + \alpha\gamma) - \gamma^2 \tag{148}$$

$$f(x) = x^2 - x(\alpha + \beta) - \alpha\beta \tag{149}$$

Can we factor these polynomials, or their difference? With how many pieces of information can we factor them such that the results are still in the irrationals?

Presumably an existant irrational factorization by a lower order quadratic would mean that the roots are shared as a part of a larger cyclic group. In this case two would be symmetric with respect to the third. This division means that the two polynomials would share a group and a factor consequently over the prime inverse radicals. If this is the case, we would like to use polynomial multiplication or division, which would be defined with different properties than the usual polynomials.

Two of the roots would behave rationally together, as compared to the rest (as they factor rationally). A product of two is a rational multiple of the third.

We can see the isomorphism is one of the rational numbers being the identity. How is the group structure of these related to that of the polynomial groups.

Consider the following group again:

$$\{\alpha = \frac{1}{\sqrt{P}}, \beta = \frac{1}{\sqrt{Q}}, \gamma = \frac{1}{\sqrt{PQ}}\} \tag{150}$$

$$\{\mathbb{Q}[\alpha], \mathbb{Q}[\beta], \mathbb{Q}[\gamma]\} \tag{151}$$

What is the group relationship between these? These overlap such that they are equivalent, but only when multiplied, it is not exactly the same as when three numbers have a least common multiple, and stay in the integers, but it is analogous. In this case, we cannot leave the rationals, so each element is its own group of numbers.

We would like to use the extra information of composites for generation of a larger basis. With rationals and a binary division, is this enough to saturate the system with rationals, sufficient to factor all composites? What is the structure of this basis?

Suppose:

$$\gamma = \alpha\beta \qquad \omega = a \cdot b \tag{152}$$

With:

$$a, b \in \mathbb{Q} \tag{153}$$

Abstractly, we would like to know how to find a or b and α or β when resolving:

$$\delta_1 = \alpha\alpha\beta = \alpha^2\beta = a\beta \tag{154}$$

$$\delta_2 = \beta\beta\alpha = \beta^2\alpha = b\alpha \tag{155}$$

As:

$$\alpha^2 \to a \in \mathbb{Q} \tag{156}$$

$$\beta^2 \to b \in \mathbb{Q} \tag{157}$$

After we take $\alpha\gamma$ and $\beta\gamma$ how do we recognize a match? We can simultaneously compare the α^2 or β^2 and divide by the magnitude of these squared to find if there exists an α or β such that when applied in reverse order:

$$\beta\delta_1 = a\beta\beta = a\beta^2 \tag{158}$$

$$\alpha\delta_2 = a\alpha\alpha = b\alpha^2 \tag{159}$$

Leaving:

$$\omega = \beta\alpha\gamma = \alpha\beta\gamma = a \cdot b \tag{160}$$

With:

$$a, b \in \mathbb{Q} \tag{161}$$

Does there exist a general way to get this process at all powers as in a modulo function? For this, we need a prime seive with inverse powers to unlimited degree.

One question we will ask is: Is there any regularity of the digits of the irrationals?

The study of prime irrational roots (inverses in $[0, 1)$) is equivalent to the study of the symmetries of polynomials and polynomial roots and root behaviors when the group structure of coefficients is examined, for small fundamental polynomials, as these roots behave cyclically as a permutation and are numbers as well as objects in a group. The symmetries of the irrational digits specifically have to do with the symmetries of these polynomials, or in the case of three, to cubics. These polynomials represent all combinations as statements.

If we can produce a group law by which certain irrationals (genuine irrationals) possess a relative pattern, that allows us to determine if they are a base in a prime or multiplied by a rational it would be interesting. These two patterns come in as a radical prime and a linear pattern with the rational primes. We would like to be able to tell when an irrational is multiplied by a rational. For this, we must multiply by known rationals that form a group. Given these could be numbers that share factors, we have common groups between them, forming a regular polygon.

12 Geometry of the Irrational Numbers

We need an answer to the question: What geometry describes the digit sequences of the irrational numbers? What groups determine these, and how do they operate? With what symmetries and what interrelationships? With what limitations and what similarity to production of numbers for groups? And, with what ramifications for the numbers in general, the rational and the real?

Part of this is to recognize the link that exists between irrational digits of a sequence and the properties of the sequence as a whole. This local to global property seems absent when we have the multiplicative property of sequence to sequence, but there is still great importance to understanding this connection.

Given we have an irrational the digits go on forever. However the question becomes, do we reach a point where the ability to differentiate a prime in a composite is obscured or lost?

The following sequence is a depiction of the development of the structure of our set:

$$\alpha \to \mathbb{Q}[\alpha] \tag{162}$$

$$\beta \in \mathbb{R} - \mathbb{Q}[\alpha] \tag{163}$$

$$\gamma \in \mathbb{R} - \mathbb{Q}[\alpha] - \mathbb{Q}[\beta] \tag{164}$$

And so on.

If our set is N elements long, or with $N \to \infty$ do we saturate $\mathbb{R}$? Or, do we need a (countable or uncountable) infinity of such sets? Can we saturate this limit with this process? One can see, this process can aid us in getting information about the structure of $\mathbb{R}$. The prime numbers satisfy this construction, and are countable, therefore we cannot saturate the reals with such a sequence. Thus structure of the irrationals is therefore truly transcendental, however, it has limitations.

There is also a notion of distance we can construct. One can form a cross ratio of points. But in what sense does this differ for irrationals? The digit space of the irrationals is a pattern, but a strange one. Here we find something interesting with the cross ratio. This has four ways to equal a rational and behaves as the product of two distances. Clearly, multiplication preserves the cross ratio. We can also use *log* for the local distance between our digit expansions.

The notion of distance is:

$$log(\frac{p}{q}\frac{r}{s}) = log(\frac{p}{q}) + log(\frac{r}{s}) \tag{165}$$

This product is also the form of a cross ratio. Since the cross ratio is preserved, so is the product of distances, if p, q, r, and s are in turn distances. With this we can see that we could have p and q or r and s both rational or irrational. The number of possibilities is the same as what we get in our logic table. This shows that for two irrationals whose difference is a rational and two rationals whose difference is a rational that the product of the distances from one irrational to a rational and the remaining irrational to a rational are a rational. This is so that the cross ratio is a rational.

An example of the cross ratio with a variable quantity is the following:

$$l = q + \frac{1}{\sqrt{5}} \tag{166}$$

$$m = \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{5}} \tag{167}$$

$$n = \frac{1}{\sqrt{3}} \tag{168}$$

$$o = q \tag{169}$$

In:

$$\chi = \frac{(l-n)(m-o)}{(m-n)(l-o)} \tag{170}$$

This results in a polynomial for the cross ratio:

$$f(q) = -5q^2 + \frac{10}{\sqrt{3}}q - \frac{2}{3} \tag{171}$$

Such that the cross ratio is zero when:

$$q = \frac{5 \pm \sqrt{15}}{5\sqrt{3}} \tag{172}$$

What if a set of three irrational numbers each carry cofactors with the others? Multiplication of any two will reveal the shared element squared, times the other number. One can picture constructing a way to find a match between a potential product of two unknowns by multiplying with these and looking for a prime or irrational factors that relate in a group structure. Can we always form these kinds of nets to discover the numbers in a composite? If we could form these, we could look for intersection among their sets, thereby arriving at a common solution.

We would also like a way to find common factors arbitrarily, so as to factor numbers. To have a three long sequence of numbers the objects must have three shared parts. So, they each share a common factor. This is with three primes effectively. This means if we have a group, we can use properties of this group, to reduce prime irrational radicals to a repeating pattern notation.

There is a difference with many. These irrationals form the vertices of a polygon in their group. One can imagine moving along the series with the motion of a triangle between them hence determined. Then, given the way that:

$$\alpha\beta \to \gamma \tag{173}$$

$$\beta\gamma \to \alpha \tag{174}$$

$$\gamma\alpha \to \beta \tag{175}$$

The notion of the irrationals of two series reflecting such that they reflect off one another in multiplication, or that they produce each other, is clear. This is a symmetry with more than a superficial resemblence to their digits. The structure is preserved with these numbers as they move through their series, indicating this is as well a local symmetry. What operation is this, that preserves this relationship between their digits, and what is the general symmetry of the irrationals?

There does not exist a unique natural isomorphism from the numbers to the sequences or vice versa. But, we can use the natural conversion isomorphism that exists with the irrationals to the rationals, to create operators that come attached to numbers with a form symbolized by the following:

$$\frac{1}{4\sqrt{37}} \tag{176}$$

Then, we take the rational part to be the program structure, and the irrational part to be the number part. Those sequences of the local variety are of the form of an α permutation and are periodic in the irrationals. While the longer full sequence is rotationally free, and one that we get by throwing away the rationals is the ω permutation.

This use of radicals indicates that for the most part we deal with operators up to the square only:

$$x^n : 0 \leq n \leq 2 \tag{177}$$

The picture revealed is of irrationals that relate to each other by groups that extend backward upon the digits, generating them as we come back from infinity, as parabolic surfaces extending through the numbers. The intersections of these parabolas generate products, fill in the space of numbers, and indicate the natural measure of the space of the digits of the irrationals. If there can be such a concept as number dimension, we would like to find the way that fractional powers relate to this.

Going back to our set:

$$\{\alpha, \beta, \gamma\} \tag{178}$$

We find the sequences produced by these to be of the form of recursion relations:

$$F[n+1] = G[n] \cdot \gamma \tag{179}$$

$$G[n+1] = F[n] \cdot \gamma \tag{180}$$

$$P[n+1] = P[n] \cdot \gamma^2 \tag{181}$$

Two numbers repeat with a period of two and one with a period of one. We can treat these like a differential equation.

A fundamental question now is: What properties of operations are preserved under a radical function?

It appears that in even powered factors being absent from the theory, these remain hidden and in doing so become points of uncertainty in the factorization. They make one non unique unless we go deeper, to a more sophisticated factorization. But this process has a limit. The rationals of a randomly chosen irrational not only obfusicate its character but appear to make the fundamental theorem of arithmetic break down with irrationals, although they are well ordered, like the natural numbers.

These radicals are just as legitamate as parts of general numbers. We only get the odd powers in the factorization so far. We need all powers of all factors for completion. We can do this with repeated factorization, if we can isolate and divide out, or multiply out, the radical, and separate it from this rational. We follow with another factorization upon the remaining irrational part. This repeated process returns all factors, if we can begin the process to start.

Given the way the irrational digits are somewhat homogeneous, and the rational discrete and regular this may be possible. We can pick up on such rational patterns, by multiplying by a sequence of rationals until we find a return to homogeniety, of minimal degree. With many numbers in the composite, we lose accuracy over the product of their digits and can no longer factor uniquely. This is what we mean by obfusication of digits. With this, prime factorization would not be unique for irrationals in general. The groups in a random irrational quickly exceed resolution to factor. Finally, given the order of the size of the infinities, a list of prime factors should be constructable, and this means countable. With uncountable infinities we clearly cannot begin a factorization on all numbers.

It appears that the answer as to where the digits in an irrational come from can be answered quite simply by examination of the groups they share with other irrationals. A class of primes can possess one characteristic with all of its members, but it may not saturate the set of reals. These digits may appear random because irrationals depend on primes for which there are no factors and they consequently possess no subgroups.

All small factor number irrational digits appear to be parametrizable by the groups of irrational numbers contained within them, even though they do not possess a regular structure. For instance one irrational among three mutual products is certainly related to the others. These digits appear to be most simply, indicative of regular sequences in the irrationals of their composite. And although not literally giving the structure of the digits, do so by way of the group structure with individual digits. A hint is given by the varieties of successive product. For the factorization problem some such sequences are:

$$\alpha\gamma\beta\gamma\alpha\gamma\beta\gamma\alpha\gamma\beta\gamma... \tag{182}$$

$$\alpha\gamma\alpha\gamma\alpha\gamma\alpha\gamma\alpha\gamma\alpha\gamma... \tag{183}$$

$$\alpha\alpha\alpha\alpha\alpha\alpha\alpha\alpha\alpha\alpha\alpha\alpha... \tag{184}$$

The digits of the irrationals are hypothesized to be a construction involving these sequences, in the way they correlate to the group structure of the prime radicals under question. It is certain that the relationship of the digits in three mutual irrationals is determined by a group law.

Irrationals are the first numbers for which we get a scaling independence of digit representation in that we always have an inhomogeneous pattern of digits. We have an endless pattern for all but one point in the whole space of irrationals, zero. The pattern fundamentally changes when we use inverse radical primes with one another in leading to a rational. The structure changes from infinite to finite in a very short interval.

We would like to for these purposes show a comparison or limit to resolution exists at a threshhold before we reach the full irrationals given by a relationship of the groups of prime factors to the partition function. These give limits on the distinguishability of irrationals.

Also of importance is the structure of the set sequences. The number of equivalently sized sets is:

$$\binom{N}{n} \tag{185}$$

The number of sets within sets without counting self loops is given by the ways to count all lengths of all sequences of ordered items and a partition. Another way to describe this is as of choices of super set and subset among N sets. This is the sum with n from 1 to N of ways to choose n among all N multiplied by the ways to choose the remaining $N-n$ among all N. for:

$$\rho_s = \sum_n \binom{N}{n} \cdot \binom{N}{N-n} \tag{186}$$

This has as a sum:

$$\rho_s = \binom{2N}{N} \approx \frac{4^N}{\sqrt{\pi N}} : N \to \infty \tag{187}$$

These are the central binomial numbers:

$$\rho_s[1] = 2, \rho_s[2] = 6, \rho_s[3] = 20, \rho_s[4] = 70, \rho_s[5] = 252, \rho_s[6] = 924... \tag{188}$$

The ratio of the number of new enclosed sets to the previous total number of enclosed sets is asymptotic to four from below.

13 Logic Revisited

The logical functions established by this paper, the logical states, and the processing of numbers are all intended to be an easy way to construct, modify, and manipulate numbers as well as interrelate and interinvolve. These are all 'softer' operations than we are used to in mathematics. The numbers can be gradually developed, meaning, we can design algorithms that slowly fill in the digits of a number.

Note that the covering complement is the operation that takes Open to Closed and vice versa, while the evaluation or truth complement is True False. We need two pieces of information for exclusion of middle thirds in true false. We need four pieces of information for exclusion of middle thirds in closed open true false. The operators therefore have a different correspondence to numbers than in binary.

A generalized function would involve using closed and open as group operators, as set expand and contract, and as quantifiers of statements in processing, as well as representatives of the group. But theoretically we have a limited set of sixteen operators. Just how general is this set and this theory?

We have shown ways of deriving self similar sequences such as:

Golden Ratio
Exponential Growth
Fibbonacci Sequence

Each successive term is a composition of those previously, such as a sucessive product, and thus many limits are possible in the broader context. We can include objects in the set of these tables to be the whole of a set of numbers. Or, even characteristic of a certain class of numbers such as prime bases. As a consequence we can do arithmetic and inferences on sequences. Can we prove that the irrational digits are given by an expression which has a property of uniqueness in the prime factorization as we can treat the natural numbers? In summary, for sequences and rings, the elements are countable and of order $\aleph_0$. For these both as well, the sets (in combination) are uncountable and of order $\aleph_1$.

14 The Circle as a Logical Fractal

The circle is a logical fractal. It has a measure of e and a dimensionality of π. It is a closed two dimensional fully symmetric figure. Here we find that geometry is now the source of logic, in that geometry is to topology as a logical system built from an instance is to all of logic. There exists a geometric language and logical system for each corresponding geometric space.

Is e, implicative of the natural measure of the base of a curved or embedded space, with π, the natural dimension of this space and with i as true, and $-i$ as false, relative to direction?

As a unique number, or prefactor for the natural dimension of this space, $i = \sqrt{-1}$ is then a continuous motion in the direction of the open space, to where the function goes tangential to itself reaching towards a form of self completion. Consider's Euler's equation:

$$e^{\pm i\pi} + 1 = 0 \tag{189}$$

This, as indicating the sense of the space, is the total volumetric degree (with no inside or outside), but, scaled and as to unity. When negative and antipodal to its natural character $e^{-i\theta}$ becomes negative. The completion of the numbers around a circle leading back to the beginning. Open to truth and logic, closed in only the geometric sense. But, traversable by motion (clockwise or counterclockwise) around the circle, π in one way, and $-\pi$ in the other way. These complete the picture and add to zero.

The circle is seen as a linear fractal with dimension: $ln(e) = 1$, due to the connection between dimension and covering. To geometry, and logic of a circular or exponential form. By asking the question: which object has an exponential ratio of 'points' to linear distance as covering, considering the form: $e^{+i*\pi} + 1 = 0$, and its equivalency to: $\pi * i = ln(-1)$. Is the role played by the circle in logic an identity as it is in geometry?

Here we find the equation for a circle literally interpreted as being the base of the geometric base of the exponential in density, to the power of a dimension of π. As a compact fractal like object with the right ratio to have an existant embedding as a circle in space. This sits at the base of geometry, logic, and number theory, as an object, the circle. With a dimension of π, a circumference of 2π, and a radius of 1, a compact space, in the circle. This is fundamental to all of mathematics.

This question is equivalent to asking; which geometry is the one for which we have fractal dimension of a space, compacted over by e another irrational measure equalling one of the ratio of scaled space to the base space? These could be the natural measures of numbers themselves.

In seeing the truthful circle as a fractal in one dimension with levels of truth, we may have closed or empty statements as unreachable, but we have no falsity over the whole space. The false one is not one of falsity and is instead the complement of this one. It is implicated by the first as existent, due to a symmetry of logic itself. It is the inversion of true and false, and the direction of logical statements. This is neither symbolic nor non symbolic.

Since there are two ways to draw a circle, it makes some sense to say there are two universal truths. If clockwise and counterclockwise refer to also the motion in a decision tree, we find that direction in one dimension can be implicative of a series reduced to its conjugate direction around the circle.

We find this also to be the balancing point where the property of the circle becomes exposed, when one asks for the geometry entirely consistent of points which has a manifestation as a curve, and there is no outside or inside. The exponential comes out as a rotation instead, neither blowing up nor collapsing, but instead meeting itself. This is manifest in the circle as a group of elements each linking to the next and containing its property of meeting only itself globally. With an infinity of true and an infinity of false we cannot ask which there are more of, True or False, and question of overall value is still open.

Taking the radius of a circle as outwards, and then perpendicularly moving around it, we find that the initial direction of the motion is perpendicular to the radius. This is a continual motion in the imaginary direction in the local coordinate system of the circle and a point thereupon. If we consider this in the more broad context, it indicates that not only is this a geometric instantiation but it also indiciates that the direction of logical flow is in the imaginary unit.

We may also begin looking for the circle by asking such simple questions as: What limit series has the property of closure, or of a recursive and wholistic (or summative) property upon the states?

With this we may come up with measures, and try to interrelate their operation with an equality, but this does take effort. Also, other functions may go to the circle.

The open everywhere statement is one of the only freely open and circular states corresponding in its behavior under the derivative like an exponential function. This helps lead towards a definition of the circle. We can search for the function that satisfies the differential identity, and then operate as a power on the logical sequence or state symbolizing e and π, but it would be better to have a relationship with meaning, within the mathematical system.

Since every statement satisfies this exponential relationship with only one base, we need to describe more complex behaviors to get the unique and non trivial strings that satisfy this identity. One way is to consider the Open and Closed rings, or the True and False strings of any order. We find that any sequence of true, and false has closed and open as the intercessor from the derivative or two concatenation, but these do not work as self similar sequences under the derivative, like the exponential.

This reveals the infinite string of truth is not just "any" sequence of true and false, when we have open and closed included as symbols with additional meaning. They acheive an interesting connection, for they yield a transformation between a string and its reversal, without a turning around of the entire string, nor balanced on a single element. This hypothetical string needs to have repetitions of Open and Closed among True and False, or, be a normal sequence.

An additional question is if the following series can account for the properties of a self similar recurrent sequence on the circle. Closed and open could refer to a shrinking or expanding of the quantity of numbers by a number density of these in a sequence. The behavior of these constants appears to be linked to the properties of the sets, instead. However, there is a real number connection to π with the sets:

$$4\sum_{k=0}^{n}\frac{(-1)^k}{2k+1} \to \pi \tag{190}$$

e appears to have to do with a natural measure of the density:

$$\lim_{n\to\infty}(1+\frac{1}{n})^n \to e \tag{191}$$

Each is produced by measures on the space of Closed, Open, True, and False. The prediction is that the sequences leading to these expressions behave and pertain to topological quantifiers, and reduce to these numbers. Are numbers that have to do with the properties of these series unique in identifying them (indicating uniqueness of series or sequence) or of identifying the properties of a similar sequence?

π, and e are natural measures of the numbers. e is that number for the amount for which it is equal to all amounts that came before. It is also the amount for which the ratio of the amount to the rate remains fixed. π is the distance to go around versus to, within this space. The measure, of the ratio of the distance that is covered to travel to a location, to exactly half the distance between these locations, when maintaining a perpendicular direction of travel the entire time to this point.

We need two conjugate quantities to form a connection. One is the number density per dimension of a number as the point. The other is of the capacity or dimension of number density filling. The first pertains to the point, the second, to the space. We use sets and elements.

Thus these definitions suggest that there is a relationship between the earlier logical sum and product as parts of the state:

$$\frac{1}{\pi} = \lim_{n\to\infty} \rho_{r,t} = \lim_{n\to\infty} \frac{1}{|t-r|} \sum_{n=r}^{t} f(n) \tag{192}$$

Where: $f(n)$ equals the number that are coprime.

$$e = \lim_{n\to\infty} \kappa_{r,t} = \lim_{n\to\infty} \prod_{n=r}^{t} f(n) \tag{193}$$

Where: $f(n)$ equals the number that are in succesive layers of sets.

e equals the ratio of subsets it contained within a set, to the number of sets that contain this set, in a given number of sets, as this number of sets goes to infinity. When we have $e^{\pm i\pi}$ this can be interpreted as the scaling of the coprimality condition on the sets. When we have i we go transverse to the set subset relationship. The limit of π gives the relative proportion that have no coprimality in a given length (the radius of the circle), to those that are coprime.

The total of two attributes scales like a hyperbolic cosine function, in that:

$$P_{w/i} + P_{w/o} = e^{-n} + e^{+n} = 2\cosh n \tag{194}$$

$$P_{w/i}/P_{total} = \frac{e^{-n}}{2\cosh n} \qquad P_{w/o}/P_{total} = \frac{e^{+n}}{2\cosh n} \tag{195}$$

This is for a property, like the scaling of coprimality. The scaling of sets with the coprime numbers as we go inwards added to the scaling of sets with coprime numbers as we go outwards, equals a total number of sets. The scaling of sets and subsets is hyperbolic when we take a viewpoint of evaluating inwards and outwards. We would need this quantity for evaluation of the total probability. We can see in this case the probability has a sigmoid like shape going from certainty to doubt or in the reverse within a $2e$ sized interval of sets and subsets. It is for one set's spread into the other spaces as a percentage of its relationships to that of the total, when we have set transitive rules. Everywhere on this construction is locally identical. We can use i to go perpendicular to the set subset relationship.

When we have the value $e^{\pm i\pi}$ it is of the form of: $density^{dimension}$. A number density to a dimension is an appropriate use of units. This makes e the density per dimension and $i\pi$ the dimension per density. The claim is that if we get a +1 for this expression we have spherical geometry. If we get -1 we have hyperbolic, and at 0, we find a flat space. e and $i\pi$ characterize two primary properties, of a special type of simple and maximal set of logical sequences. What holds for these sequences, which have the density in the reals according to the way they scale, holds for the reals.

We find: π as the inverse of the limit of the average of the proportion of total sets to those for which commensurability in the modular sequence is attained. This is the percentage of n among modular sets for which the system is commensurate for both the distances n and $N - n + 1$ in terms of equal total sets covered. This is the inverse of the percentage of the number that have agreement conditions among all modular sets of two. This number is generated non-combinatorily.

The number of modular groups in total is simple to tally. It is the sum of N identical objects in all group sizes:

$$\frac{N(N+1)}{2} \tag{196}$$

The commensurability occurs when the number of periodic multiples of a given length occur exactly once on all of the elements of the group. This is when their least common multiples divided by their length into a common distance are equal. Numbers are coprime for a length L, with a division at m:

$$\gcd(m^n - 1, m^N - 1) = m^{\gcd(n,N)} - 1 \tag{197}$$

For these numbers:

$$\frac{LCM(n,N)}{n} = \frac{LCM(N-n+1,N)}{N-n+1} \tag{198}$$

The probability that converges to $1/\pi$ is the probability that the two distances n and $N-n$ are mutually coprime to N, or that the lengths cover the same total number of distances commensurably.

In π, we have the ratio of the total combinatorial number of sets to the number of sets with the property of commensurability in a given length. The length of this problem going to infinity, as it was arbitrary, admits a natural congruence relation between randomly lengthed lengths.

They become commensurate with this likewise probability never if they are non coprime. These are the rationals in non-reduced form. If they are congruent from coprimality in a given length they produce two rationals when divided into this length. Hence, the notion of an irrational being relative to a given unit length, finds creedance. We find that the proportion of those that go on forever (non-congruent and non-reduced) to ones that terminate and are congruent (reduced rationals) to a given distance is:

$$1 - \frac{1}{\pi} : \frac{1}{\pi} \tag{199}$$

But the density of rationals is constant, so this holds for rationals in general at all lengths, however, it may not hold for infinity. This ratio is $\pi - 1$, approximately: 2.14159265, and it's inverse is approximately: 0.466942207. If we multiply by π further we get: 1.466942207, which is one plus this ratio. This is an anharmonic ratio of π. We find this also with the sets of sets. The ratio of the number of with to without an agreement or commensurability condition among combinations, is in agreement with this result.

We also find that among sets with subsets the probability of with to without is found to be:

$$P_{w/,w/o} = \frac{\frac{1}{\pi}}{1-\frac{1}{\pi}} = \phi \tag{200}$$

With the property of commensurability to the total it is:

$$P_{w/,total} = \frac{1}{\pi} \tag{201}$$

Dividing, we find:

$$P_{w/o,total} = 1 - \frac{1}{\pi} \tag{202}$$

This makes for a total probability of:

$$P_{total} = 1 \tag{203}$$

This means there are:

$$P_{w/} = \frac{1}{\pi} \tag{204}$$

Solutions to:

$$\frac{LCM(n,N)}{n} = \frac{LCM(N-n+1,N)}{N-n+1} \tag{205}$$

At a size of N.

We have found the probability of two of the same elements both being coprime to a third. We can use this information to build a network of relationships between sets with known probabilities in such a way that we can predict the likelihood of a given element in a set of numbers to be coprime or prime. We can place bounds on the probability due to the concept of inclusion and exclusion in groups, under combinations between their elements. The ratios of classes of numbers can be probabilistically narrowed down to incredible levels.

We also find: e is equal to the limit of the modular sequences as length $N-n$ divided by a length of N and taken to a power equal to N. e corresponds to the limiting ratio of length remaining to the power of the total length as the distance goes to infinity. This can be interpreted as a measure of the scaling of the probability as n over an infinity of sets when the size of these sets: N goes to infinity. This happens naturally in our strings, because there are N sets within each of the N sets.

It is the infinite limit, of the rate of a mutual condition of contained and contains holding for all N upon all N, when it holds for one. It holds at intervals of n, compared with N sets. Thus, the ratio for sets inside another set to the number of these sets, for a function on every n elements is e in the limit of an infinite number of sets for all n. This is an exponential distribution over the sets. Thus the exponential logical function is a modular function with uniform set values from the uniform ring of all open to true and false. The lengths are:

$$x : y - x : y \tag{206}$$

On dividing by y to give the difference to the whole as one:

$$\frac{x}{y} : 1 - \frac{x}{y} : 1 \tag{207}$$

Taking the power of y if $1 - \frac{x}{y}$ is interpreted as a probability of one among a set gives the probability of :

$$(\frac{x}{y})^y : (1 - \frac{x}{y})^y : 1 \tag{208}$$

We can see that the limit of $y \to \infty$ is the exponential constant, e to a power of $-x$. Specifically, e is the limit of the ratio of a difference between a total length and a smaller length, to the power of the total length going to infinity Under a reflection at $N/2$ the behavior is symmetric across the two halves of the real number interval from 0 to 1. Hence, this is a re-expression of the other side under this reflection.

This is the ratio of number density to number dimension. For example a dimension of 0 yields $e^0 = 1$. For one point in zero dimensions. But logic, has an imaginary argument of i. We get a power of π from the probability ratio of total numbers to those with the coprimality. This is the space occupied by objects that don't behave like primes, in that they have factors with other numbers, in the larger space.

This is more a way to measure relative rates of growth than numbers specifically. We find that the probability of the numbers being non-coprime with this number going to infinity, gives instead the proportion that divide one another. Thus the smaller length represents those that are coprime. The probability of non coprimality to coprimality, is ϕ.

If we take the tones at all lengths, the inverse of the proportion for which the introduction of a node produces standing waves is π, for these numbers, which include all rationals. What this says in sum, is that the inverse of the proportion of modular set conditions that are satisfied, taken as the distance x with the remaining distance $y - x$, is maximally harmonic with the total distance. To obtain this result we take the length and the size of the sequence interval to infinity, and in doing so find the ratio of numbers that share a factor (non coprime rationals) to those that do not share factors (the coprime rationals).

The exponential, is the limiting probability for a given lengthed remainder, (thus, a set rule), divided by the total length, and taken to the power of this length, (or the number of sets). It is the power by which probability comparatively wanes as we move to subsets of sets, versus these sets.

This is a measure of how many tones divided into two pieces would be in harmony, compared to unity, as taken at all length scales and in the limit of the number of refinements or gradations going to infinity. These are the harmonies of the circle: the scaling of the number of tones and the proportions of those that do not divide into one another. This constitutes the harmonic modes. With these, e and π, we have unveiled the natural structure of the circle with the reals.

The harmony of modes occurs over the tones that share no fundamental tones, counted in the denominator. If our set consists of all tones, then we have a fundamental ratio of π between these two kinds. This is thus the weight of tones which carry harmony with some others to the ones which do not.

We can now re-interpret the formula:

$$e^{\pm i\pi} + 1 = 0 \tag{209}$$

This is the limiting natural measure of the scaling of numbers of subsets to sets (e), to the power of the limiting natural measure of the proportion that share a period in periodic coverings among sets (π). π is the ratio of open periodicity to incommensurate tones among the number sets. This is the ratio of all periodic coverings to those with coprime periods summing to the number of sets. When the distance is repeated, these coprime ones are the tones that are commensurate in a length. This is thus the scaling of the periodicity of modes as we move from set to subset.

With i we move among sets on the same level. If we were to only have sets that share modes with the other sets on the same level or that do not contain them, then given only the coprime modes have a meeting condition, the proportion of these that do not cover all of the set period to those that do is a ratio of π. The scaling of periodicity with this has a fixed magnitude of one, indicating the ratio of periods that are incommunsurate to commensurate and are periodic in some period q, scale equally in number proportion **with** these periods as we move to adjacent sets and larger numbers.

This holds for the rational numbers. We have not yet clarified the irrationals. We would not expect periodicity in an irrational because numbers are equiuniformly representative of their values geometrically. With respect to the natural numbers they are of a uniform rate as well. Hence, for an irrational to repeat, we would need some numbers that behave differently. For example, the squares of a number used in both counting and multiplication. This would mean they would show nonlinearity. These would show a squeezing of the domain over which a number is a defined, such that we would have a mapping of more than one element to some squares. If they are equiuniform we find the counter intuitive result that a number can contain part rationality and part irrationality.

Should the quality of irrationality be dictated by the digits or the algebraic representation? If it is dictated by the algebraic representation then we have a better definition of what rational and irrational means. If one is not rational it cannot be put in such a correspondence with two integers. But these exactly define our rings with periodic sets. There is however a way to get an irrational number. Consider the amount of space occupied by the periodicity. These constitute a countable set. If the remaining part (which is uncountable in comparison) is of the structure of the $\mathbb{R}$ then these in an average or sequence evaluation can individually result in an $\mathbb{R}$ result, for they are in number $\aleph_1$.

The sequences are countable, however they generate all subsets of all sets, the full cardinality of which is that of the reals. If we begin at any of of N positions and have unidirectionality, or we consider the two directions as equally important, then we find that the cardinality can be computed as $4^{\aleph_0} = \aleph_1$. These are therefore in number $\aleph_1$ in comparison to the full set of sets of countability $\aleph_0$. Therefore although these are not digits, all pieces of the construction are relevant to the construction of the number. We can induce a correspondence by application of a ruleset to a string. This is merely to construct an irrational number. One such result has already been obtained, although it is algebraic, Φ, the golden ratio.

There are two distinct interpretations of numbers on the rings. With a countable infinity of natural numbers for the sets in a circle, and another countable infinity of set relationships for every set, we have two distinct ways to number. Clearly, we can make the correspondence to a power in a base arithmetic with a set depth, with the value being the coefficient, thus creating a number representation. Our two ways of reading off a number's coefficients change, and take on different interpretations if we read them off in successively contained sets or within one set. This is the periodic and aperiodic way.

The set exists in one region of the space, and as an interval, although infinitely thin, is only on one part of the ring. This is the aperiodic regime of behavior. When we instead choose a periodic sequence we get a different series of digits. In this construction what is the number where we get the same sequence of digits? What properties chacterize this aperiodicity?

15 Conclusion

It has taken us a great deal to get here, but we have covered a lot of ground.

Every modular structure that is enumerable admits a commensurate interval among several other complimentary ones. Because these admit a logical structure that ends or terminates in a modular free base. This means by the control of the base of the number system of this number, we arrive at two complimentary lengths, which may be smoothly mapped to π and e. These are then derivational of all numbers and they definitional of them. By π and e we have a number coordinate chart. What is the consistuative law of this space? Where are the unities, or identities?